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Rheology of Liquids and Solids

Modern Construction Materials – Lecture 10
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Rheology

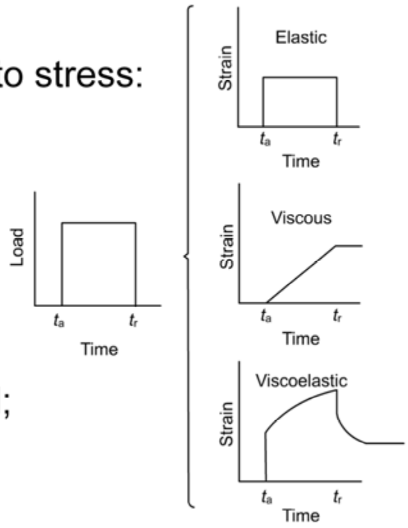
- Rheology is the science of the deformation and flow of materials.
- Rheology concerns the time-dependence behaviour of both solids and liquids.

Time-Dependent Material Response

Types of time-dependent response to stress:

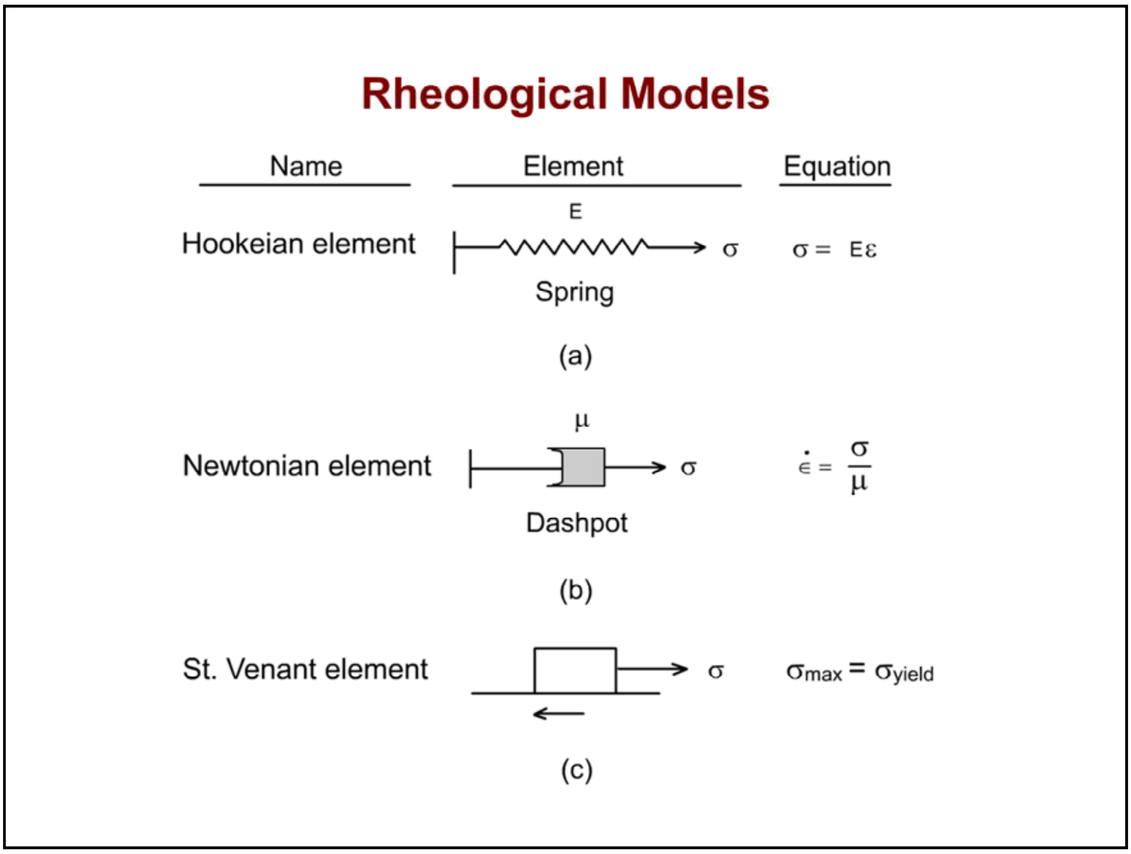
- **Elastic**: All strain is instantaneous; when load is removed, all strain is recovered.

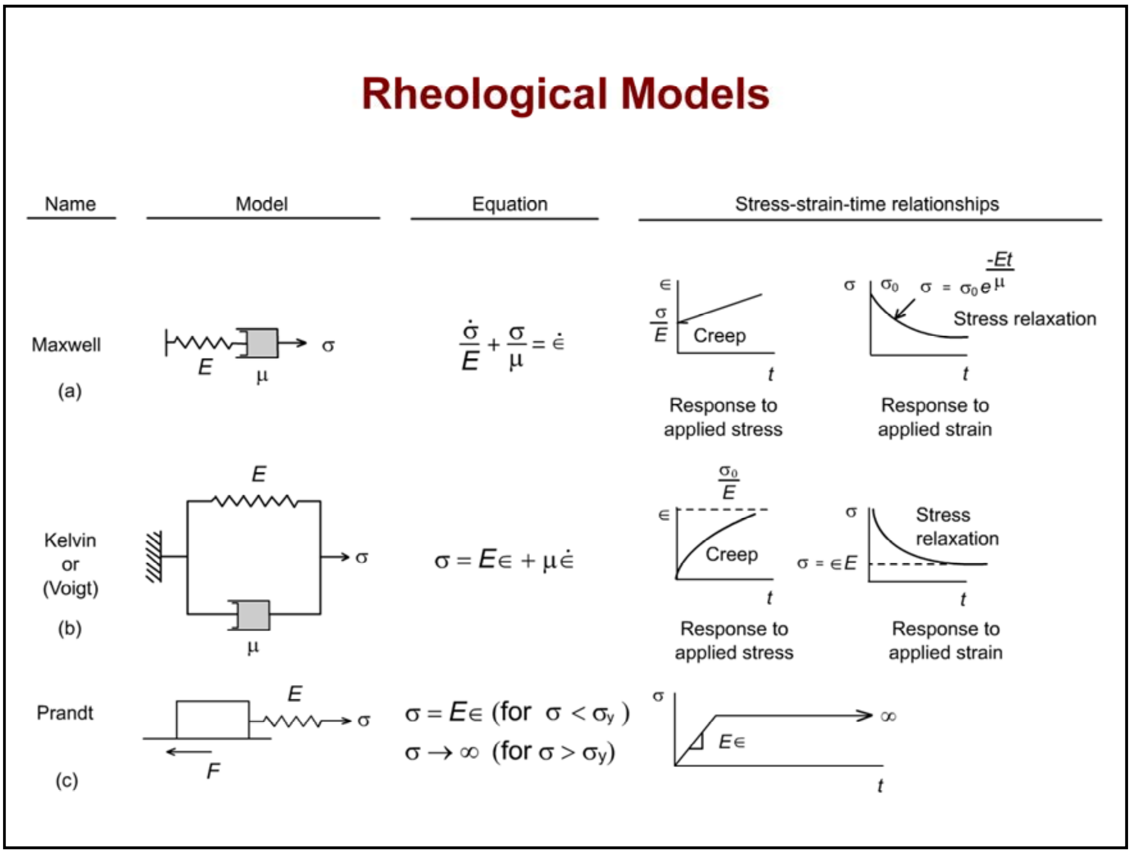
- **Viscous**: The strain increases continuously with time under load; the strain is not recoverable.



- **Viscoelastic**: There is an instantaneous strain when load is applied and the strain increases with time under load; the strain is partially recoverable.

Young et al.


Rheological Models
Name
Element
Equation
Hookeian element
E
σ
Spring
$\sigma = E\varepsilon$
(a)
Newtonian element
μ
σ
Dashpot
$\dot{\epsilon} = \frac{\sigma}{\mu}$
(b)
St. Venant element
σ
$\sigma_{max} = \sigma_{yield}$
(c)


Rheological Models
Name
Model
Equation
Stress-strain-time relationships
Maxwell
(a)
E
μ
σ
$\frac{\dot{\sigma}}{E} + \frac{\sigma}{\mu} = \dot{\epsilon}$
∈
σ/E
Creep
t
Response to applied stress
σ
σ0
$\sigma = \sigma_0 e^{\frac{-Et}{\mu}}$
Stress relaxation
t
Response to applied strain
Kelvin or (Voigt)
(b)
E
μ
σ
$\sigma = E\epsilon + \mu\dot{\epsilon}$
σ0/E
∈
Creep
t
Response to applied stress
σ
Stress relaxation
σ = ∈E
t
Response to applied strain
Prandt
(c)
E
σ
F
$\sigma = E\epsilon$ (for $\sigma < \sigma_y$)
$\sigma \to \infty$ (for $\sigma > \sigma_y$)
σ
E∈
∞
t

Rheological Models

Viscoelastic material behaviour is modelled by combining three basic mechanical elements:

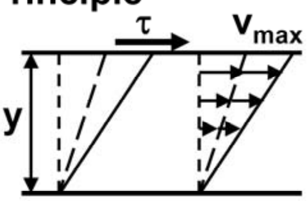
- Hookeian element (spring), perfectly elastic, energy stored as strain energy.
- Newtonian element (dashpot), viscous, all energy is dissipated.
- Saint Venant element, represents yield strength which is time-independent.

Young et al.

Rheological Behaviour of Liquids

Rheometer or Viscometer

Principle



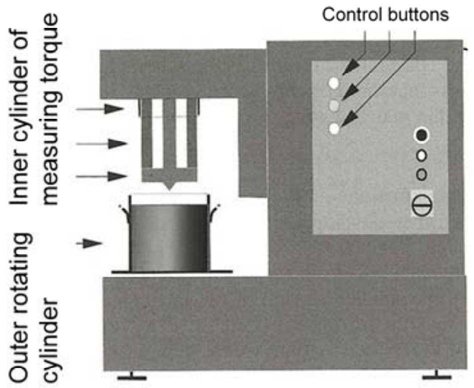
dv/dy = tan $y \approx \dot{\gamma}$

$\dot{\gamma}$ = shear strain rate
(or velocity gradient)

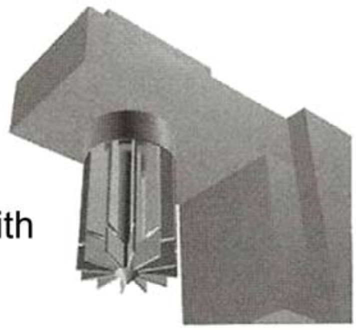
τ = shear stress

Rheological Behaviour of Liquids

Schematic diagram of the BML-Viscometer



The inner cylinder with measuring unit



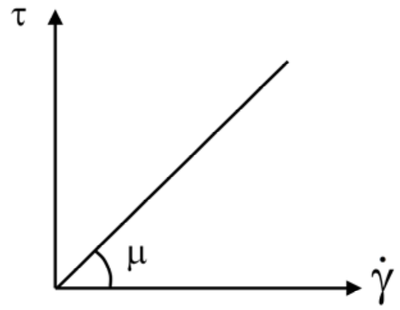
Bartos et al.

Viscous Behaviour of Liquids

Newton's Law of Viscosity: $\boxed{\tau = \mu\dot{\gamma}}$

Newtonian fluids: Fluids which obey Newton's law: Shearing stress is linearly related to the rate of shearing strain.

Viscous Behaviour of Liquids



- This response is called a ***flow curve.***
- The proportionality constant is the ***viscosity.***
- The viscosity of a fluid represents its resistance to flow under an applied shear stress.

Viscous Behaviour of Liquids

- The unit of viscosity (or dynamic viscosity or coefficient of viscosity) is: [force × time / area] – [N·s/m^2], [Pa·s] or [kg/m/s]. Also, poise [P] = 0.1 kg/m/s.

- Typical values at 20°C and atm. pressure:
  water: 1.00×10^{-3} kg/m/s or 1.00 cP;
  air: 1.82×10^{-5} kg/m/s or 0.0182 cP.

Douglas et al.

Viscous Behaviour of Liquids

The viscosity of a liquid depends on temperature (T) and pressure (p):

$$\mu(T,p) = \mu_0 A_1 \exp\left\{A_2\left(\frac{1}{T}-\frac{1}{T_0}\right)\right\}\exp\left\{A_3\left(p-p_0\right)\right\}$$

where μ_o: viscosity at T_o and p_o (reference temperature and pressure)

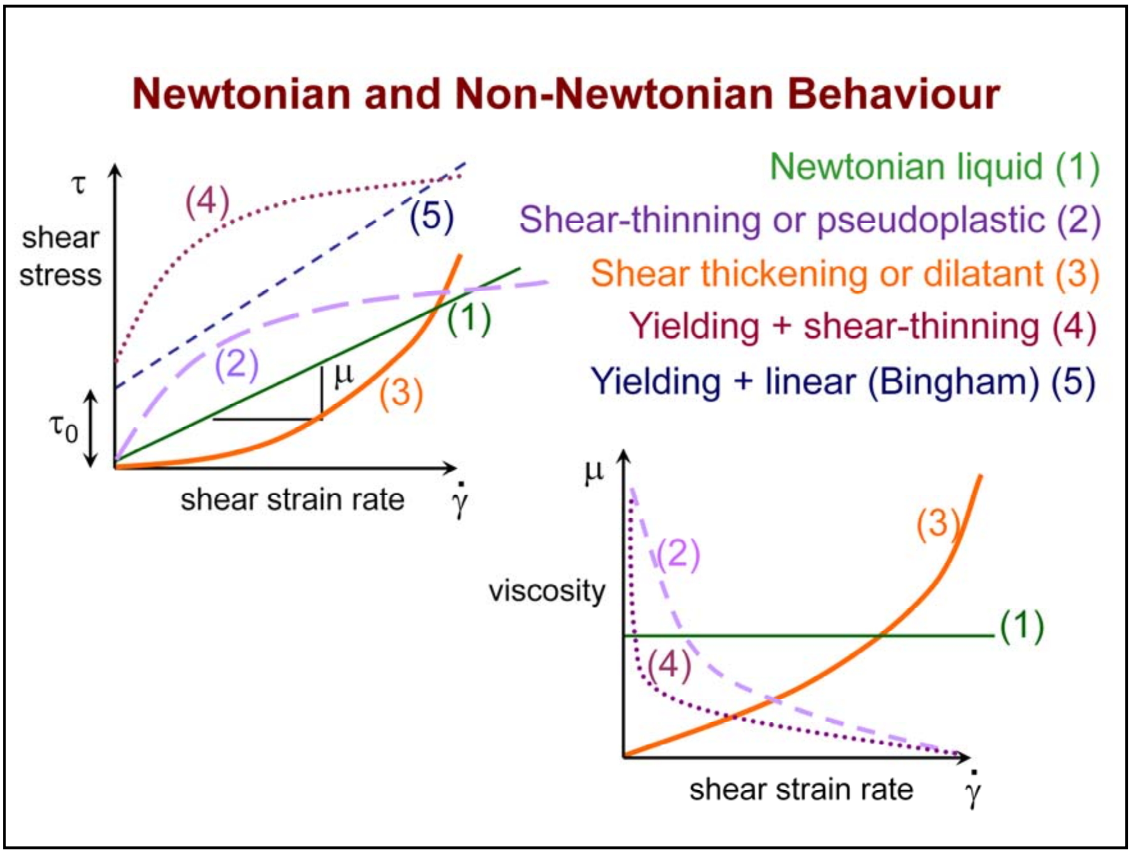
A_1, A_2, A_3: material constants

Viscous Behaviour of Liquids

The viscosity of a liquid decreases as temperature increases, and increases at higher pressures. For example, the viscosity of water goes from 1.79 cP to 0.28 cP in the temperature range from 0°C to 100°C; it doubles when the pressure increases from 1 to 1000 atm.

(The viscosity of a gas generally increases with an increase in temperature due to higher frequency of intermolecular collisions.)

Douglas et al.

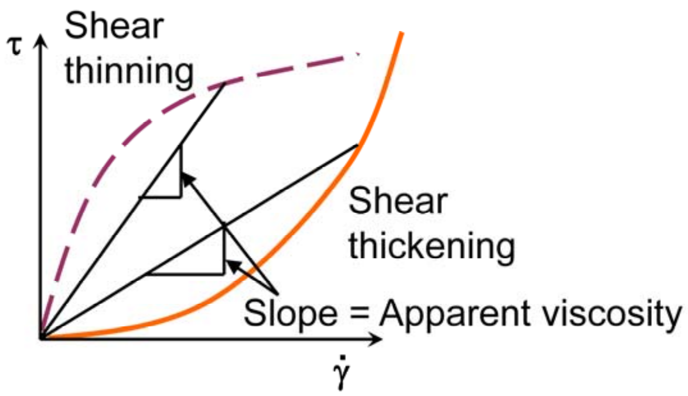

Newtonian and Non-Newtonian Behaviour
Newtonian liquid (1)
Shear-thinning or pseudoplastic (2)
Shear thickening or dilatant (3)
Yielding + shear-thinning (4)
Yielding + linear (Bingham) (5)
τ
shear
stress
τ0
μ
shear strain rate
γ̇
viscosity
(1)
(2)
(3)
(4)
(5)

Newtonian and Non-Newtonian Behaviour

- Newtonian liquid (1): $\tau - \dot{\gamma}$ curve is linear; viscosity μ is a constant $\{\neq f(\dot{\gamma})\}$
- Non-Newtonian liquids: $\mu = f(\dot{\gamma})$
  - Shear-thinning or pseudoplastic (2)
  - Shear thickening or dilatant (3)
  - Yielding + shear-thinning (4): shear-thinning with yield stress τ_0
  - Yielding + linear (Bingham) (5): $\tau = \tau_0 + \mu_p \dot{\gamma}$

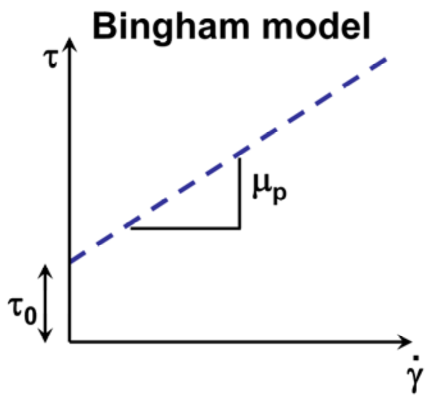
Plastic

Terminology in Non-Newtonian Behaviour



When the viscosity of a non-Newtonian liquid is determined on the basis of a single shear strain rate, it should be denoted as the apparent viscosity.

Terminology in Non-Newtonian Behaviour

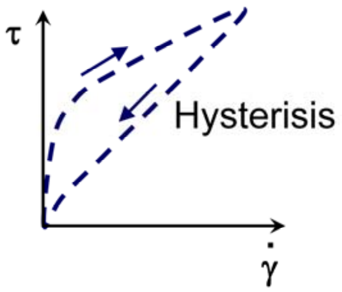


τ_0 = yield (shear) stress

μ_p = plastic viscosity

These are known as the Bingham parameters

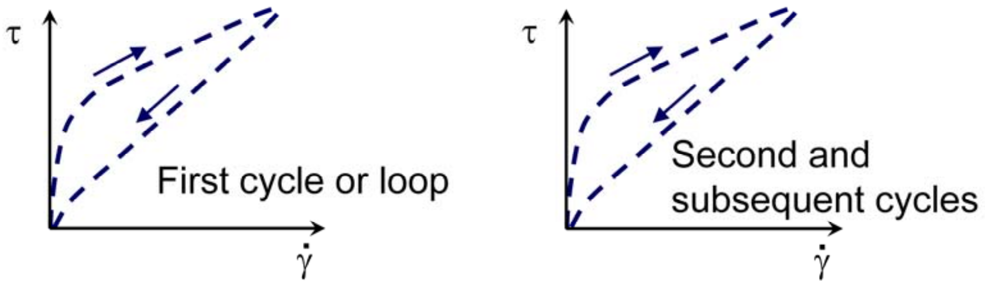
Hysterisis and Thixotropy



- Shear thinning liquids exhibit hysterisis loops.
- Increasing shear leads to a gradual breakdown of the flocculated structure, especially in particulate suspensions.
- If this breakdown is maintained, shear stress needed in the downward branch for the same strain rate is lower.

Hysterisis and Thixotropy

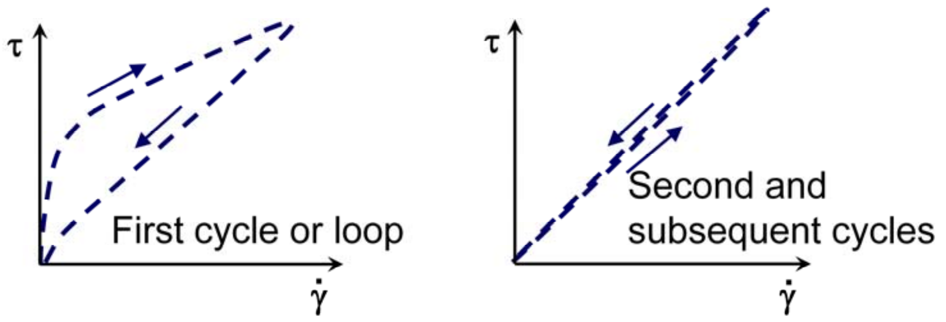
Thixotropic



- When subsequent loops are similar to the first, the material is said to be thixotropic.
- The bonds re-form when the fluid is at rest, and the response is the same in the next cycle.

Hysterisis and Thixotropy

Non-thixotropic



- In a <u>non-thixotropic</u> fluid, the second and subsequent cycles will be identical to the descending branch of the first cycle.

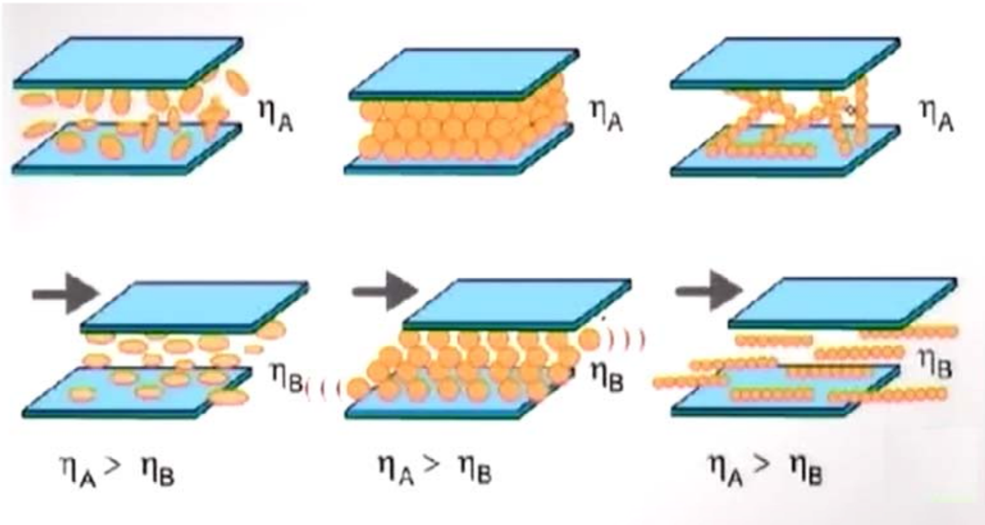
Young et al.

Shear-Thinning in Suspensions

In colloidal suspensions, shear-thinning can occur due to:

- Alignment of initially randomly oriented particles to offer less resistance to flow
- Dispersion of initially closely packed particles
- Separation and alignment of chain molecules and long particles that are initially in three-dimensional network

Shear-Thinning in Suspensions



Examples

- Newtonian fluids: gases such as air, and liquids such as water, petrol and light oils.
- Shear thinning: polymer melts such as molten polystyrene, liquids with fine suspended solids (slurries & pastes), and some paints.
- Shear thickening: corn starch, some slurries and some solutions of certain surfactants.
- Bingham: fresh cement pastes and concretes
- Thixotropic fluids: Bentonite-water suspensions, asphalts, epoxies and adhesives.

References

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- *Workability and Rheology of Fresh Concrete: Compendium of tests*, Eds. P.J.M. Bartos et al., RILEM Publ., Bagneux, France, 2002