



IIT KHARAGPUR



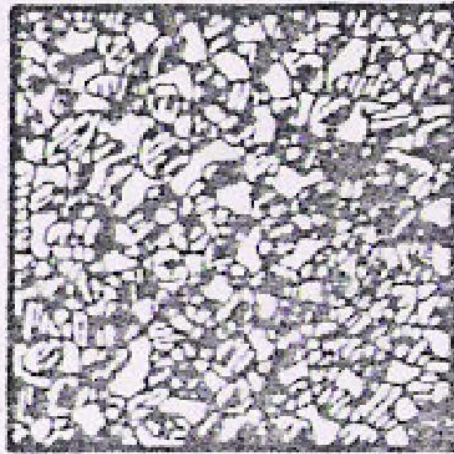
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CERTIFICATION COURSES

SOIL MECHANICS/GEOTECHNICAL ENGINEERING I

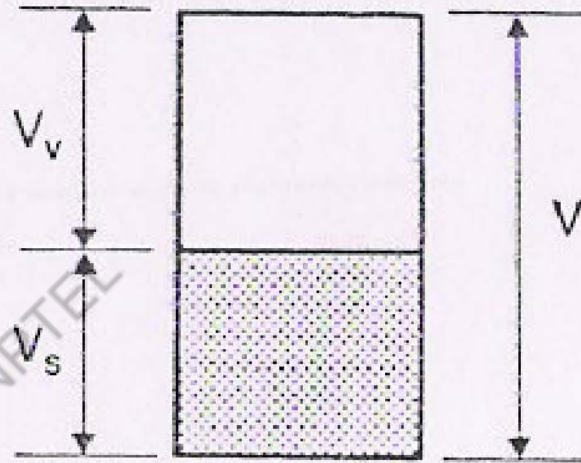
THREE-PHASE DIAGRAM

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IIT KHARAGPUR

THREE-PHASE DIAGRAM



(a) Actual form



(b) Idealised form

THREE-PHASE DIAGRAM

$$\text{Void ratio, } e = \frac{\text{volume of voids}}{\text{volume of solids}} = \frac{V_v}{V_s}$$

Range 0 to ∞

$$\text{Porosity, } n = \frac{\text{Volume of Voids}}{\text{Total Volume}} = \frac{V_v}{V} = \frac{V_v}{V_v + V_s} = \frac{e}{1 + e}$$

Range 0 to 1

THREE-PHASE DIAGRAM

The specific gravity of a material is the ratio of the weight or mass of a volume of the material to the weight or mass of an equal volume of water. In soil mechanics the most important specific gravity is that of the actual soil grains and is given by, G_s

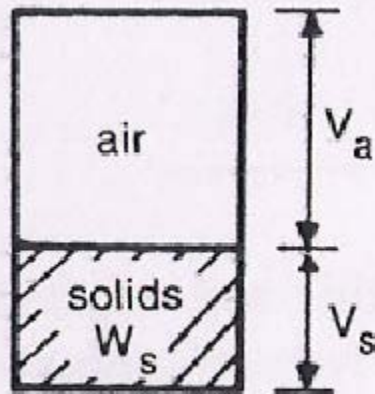
From the above definition it is seen that for a soil sample with volume of solids, V_s and weight of solids, W_s

$$G_s = \frac{W_s}{V_s \gamma_w} = \frac{M_s}{V_s \rho_w} \quad \rho_w = 1.0 \frac{\text{gm}}{\text{cm}^3} = 1.0 \frac{\text{Mg}}{\text{m}^3}$$

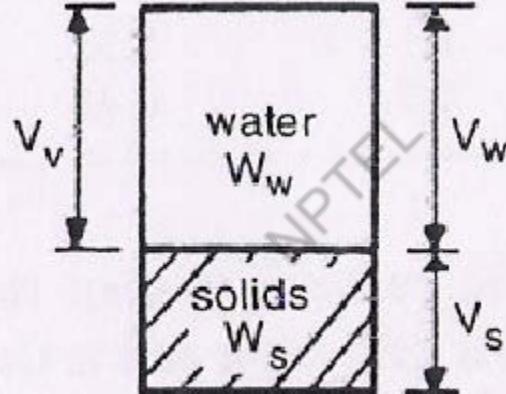
Specific gravity of most soil
range between 2.6 and 2.75

$$\gamma_w = \rho_w g \quad g = \text{acceleration due to gravity} = 9.81 \text{ m/s}^2$$

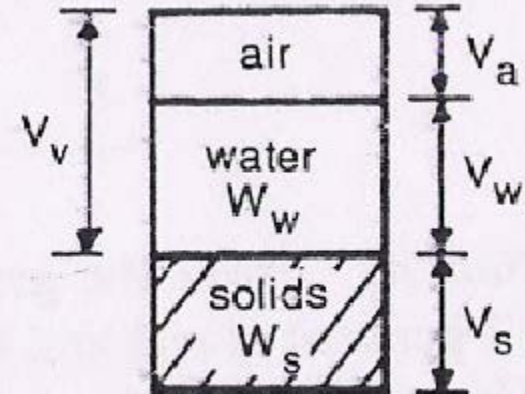
THREE-PHASE DIAGRAM



(a) Dry soil



(b) Saturated soil



(c) Partially saturated soil



THREE-PHASE DIAGRAM

$$\text{Degree of saturation, } S_r = \frac{\text{Volume of water}}{\text{Volume of voids}} = \frac{V_w}{V_v}$$

Range 0 to 100%, Degree of saturation is zero for completely dry soil

Degree of saturation is 100% for completely saturated soil

THREE-PHASE DIAGRAM

$$\text{Density, } \rho = \frac{\text{mass}}{\text{volume}} = \frac{M}{V} \quad \text{Unit kg/m}^3$$

$$\text{Unitweight, } \gamma = \frac{\text{Weight}}{\text{Volume}} = \frac{W}{V} \quad \text{Unit kN/m}^3$$

Density of water 1000 kg/m³,
Wt of 1000 kg mass is 1000x9.81 N
Hence, Unit weight of water = 9.81 kN/m³

THREE-PHASE DIAGRAM

$$\begin{aligned}\text{bulk unit weight, } \gamma_{bulk} &= \frac{\text{total weight}}{\text{total volume}} = \frac{W}{V} = \frac{W_s + W_w}{V_s + V_v} \\ &= \frac{G_s V_s \gamma_w + V_v \gamma_w S_r}{V_s + V_v} = \frac{(G_s + S_r e) \gamma_w}{1 + e}\end{aligned}$$

$$\text{Saturated unit weight, } \gamma_{sat} = \frac{\text{Saturated weight}}{\text{total volume}}$$

$$\gamma_{sat} = \frac{G_s + e}{1 + e} \gamma_w$$

When soil is saturated,
 $S_r = 1.0$

THREE-PHASE DIAGRAM

Dry unit weight, γ_d

$$\gamma_d = \frac{\text{Dry weight}}{\text{total volume}} = \frac{G_s \gamma_w}{1 + e} \quad S_r = 0.0 \text{ when completely dry}$$

Buoyant unit weight = saturated unit weight – unit weight of water

$$= \frac{(G_s + e)}{1 + e} \gamma_w - \gamma_w = \frac{G_s - 1}{1 + e} \gamma_w$$

THREE-PHASE DIAGRAM

Relationship between w , γ_d and γ_{bulk}

$$\gamma_{bulk} = \frac{W_w + W_s}{V}, \quad \gamma_d = \frac{W_s}{V}$$

$$w = \frac{W_w}{W_s} \text{ or } W_w = wW_s$$

$$\gamma_{bulk} = \frac{W_s}{V} (1 + w) \text{ or } \gamma_d = \frac{\gamma_{bulk}}{(1 + w)}$$

THREE-PHASE DIAGRAM

Relationship between e , w , G_s and S_r

$$w = \frac{W_w}{W_s} = \frac{V_w \gamma_w}{V_s \gamma_w G_s} = \frac{V_w}{V_s G_s} = \frac{S_r e}{G_s}$$

$$S_r e = w G_s$$

For saturated soil, $S_r = 1.0$,

$$e = w G_s$$

THREE-PHASE DIAGRAM

Air content: $n_a = \frac{V_a}{V}$

From Three-phase diagram

$$V = V_s + V_w + V_a \quad \longrightarrow \quad 1 = \frac{V_s}{V} + \frac{V_w}{V} + n_a \quad \longrightarrow \quad 1 - n_a = \frac{V_s}{V} + \frac{V_w}{V}$$

$$1 - n_a = \frac{V_s}{V} + \frac{V_w}{V} = \frac{W_s / G_s \gamma_w}{V} + \frac{W_w / \gamma_w}{V} = \frac{\gamma_d}{G_s \gamma_w} + \frac{w W_s / \gamma_w}{V} = \frac{\gamma_d}{G_s \gamma_w} + \frac{w \gamma_d}{\gamma_w}$$

$$1 - n = \frac{\gamma_d}{\gamma_w} \left(w + \frac{1}{G_s} \right) \quad \longrightarrow \quad \gamma_d = \frac{(1 - n_a) G_s \gamma_w}{1 + w G_s}$$

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THREE-PHASE DIAGRAM

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TREE-PHASE DIAGRAM: Numerical problems

1. A moist soil sample weighs 346 g. After drying at 105°C its weight is 284 g. The specific gravity of the mass and of the solids is 1.86 and 2.70, respectively. Determine (a) The water content (b) The void ratio and (c) the degree of saturation.

TREE-PHASE DIAGRAM: Numerical problems

2. A soil deposit is being considered as a fill for a building site. In its original state in the borrow pit the void ratio is 0.95. Based on the laboratory tests, the desired void ratio in its compacted state at the building site is to be no greater than 0.65. Determine percentage decrease of volume of the deposit from its original state.

TREE-PHASE DIAGRAM: Numerical problems

3. A soil sample taken from a borrow pit has an in situ void ratio of 1.15. The soil is to be used for a compaction project where a total of $100\,000\text{ m}^3$ is needed in a compacted state with the void ratio predetermined to be 0.73. Determine how much volume is to be excavated from the borrow pit.

TREE-PHASE DIAGRAM: Numerical problems

4. A soil sample in its natural state has, when fully saturated, a water content of 32.5%. Determine the void ratio, dry and total unit weights. Calculate the total weight of water required to saturate a soil mass of volume 10 m^3 .

TREE-PHASE DIAGRAM: Numerical problems

5. Material for an earth fill was available from three different borrow sites. In the compacted state the fill measured $100,000 \text{ m}^3$ at a void ratio of 0.70. The corresponding in-situ void ratio and cost (material and transportation) of the material for three sites is as follows:

Borrow site	Void ratio	Total cost per cubic meter
1	0.8	Rs 200/-
2	1.7	Rs 180/-
3	1.2	Rs 160/-

Determine the most economical site

TREE-PHASE DIAGRAM: Numerical problems

6. Two sites are being considered for 'borrow' soil. The in situ unit weight of the soil of the first site is 16.2 kN/m^3 , the water content was found to be 10%. On the second site the unit weight of the soil was found to be 15.4 kN/m^3 and its water content 14%. The construction site requires 26500 m^3 of soil in a compacted state, at a unit of 18.75 kN/m^3 , at a water content of 14%. The soil from the one site required additional water. The cost for the 'borrow' material was based on volumes of soil removed from the respective sites. That is, more volume would have to be removed from the site that had 15.4 kN/m^3 weight than from the other one. The unit price from each site was Rs. 100/- for the material and Rs. 125/- for transportation, for the respective cubic meter. In addition, for the material that required the additional water, Rs. 15/- per cubic meter was estimated to be the additional cost. Determine the cost of material from each site.

Thank You!!





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SOIL MECHANICS/GEOTECHNICAL ENGINEERING I

PERMEABILITY AND SEEPAGE

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Permeability and Seepage

Subsurface Water: All water found beneath the earth's surface... The main source of subsurface water is rainfall, which percolates downwards to fill up the voids and interstices. Water can penetrate to a considerable depth, estimated to be as much as 12000 meters.. Below this level water can not exist in a free state, although it is often found in chemical combination with the rock minerals.

Permeability and Seepage

Subsurface can be split in to two distinct zones:

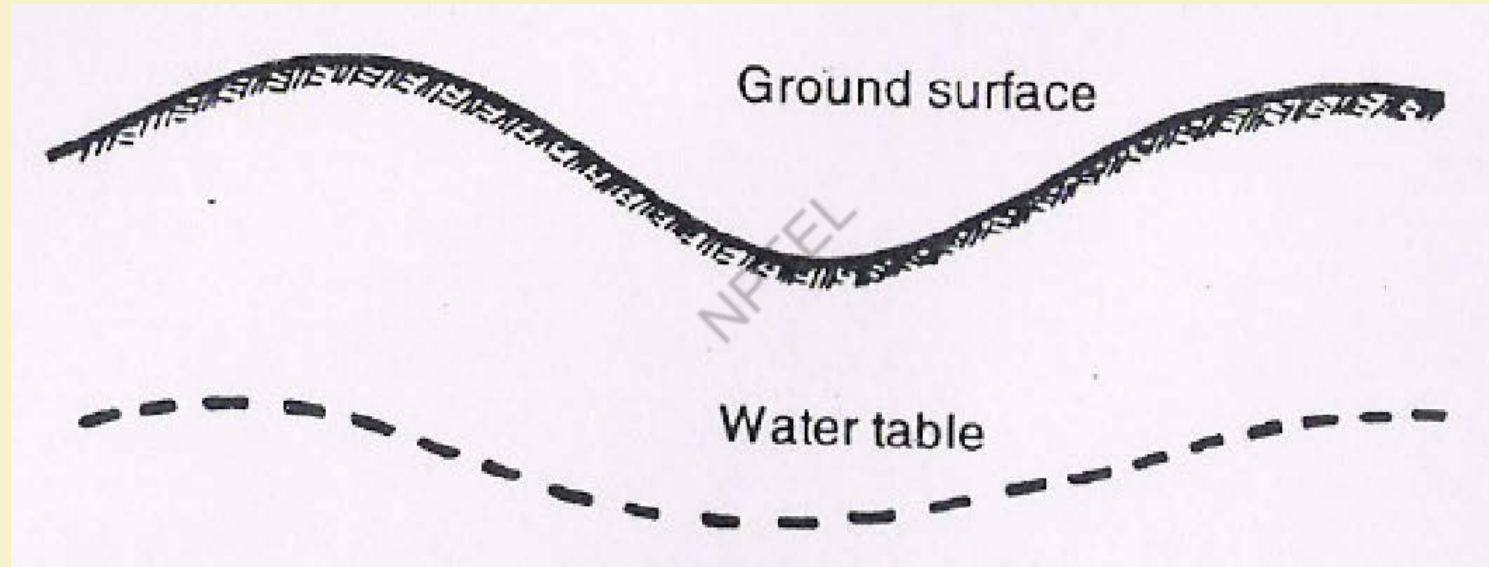
Saturation Zone: This is the depth throughout which all the fissures, void spaces, etc are filled with water under hydrostatic pressure. The upper level of this water is known as the water table, phreatic surface or ground water level, and water within this zone is called phreatic water or ground water.

Permeability and Seepage

The water table tends to follow in more gentle manner the topographical features of the surface. At ground water level the hydrostatic pressure is zero, so another definition of water table is the level to which water will eventually rise in an unlined borehole.

The water table is not constant but rises and falls with variation of rainfall, atmospheric pressure, temperature etc., whilst coastal regions are affected by tides. When water table reaches the surface, springs, lakes, swamps and similar features can be found.

Permeability and Seepage



Permeability and Seepage

Aeration zone: This zone occurs between the water table and the surface, and can be split into three sections.

Permeability and Seepage

Capillary Fringe: owing to capillarity, water is drawn up above the water table into the interstices of the soil or rock. Water held in this manner is in a state of suction or negative pressures; its height depends upon the material, and in general the finer the voids the greater the capillary rise. In silts the rise can be as high as two and a half meters and in clays can reach twice that amount, as illustrated later.

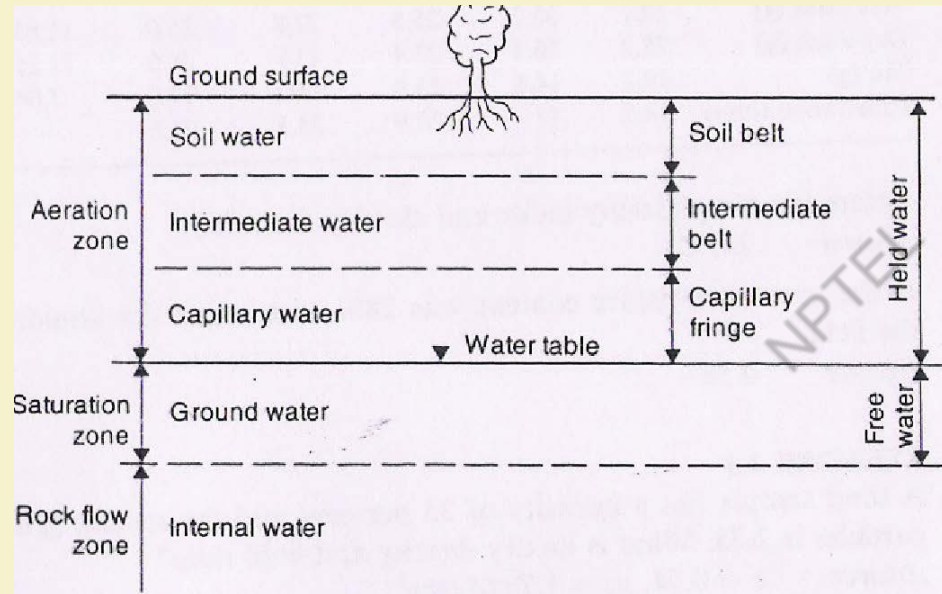
Permeability and Seepage

Intermediate belt: As rainwater percolates downward to the water table a certain amount is held in the soil by the action of surface tension, capillarity, adsorption, chemical action etc. The water retained in this manner is termed held water and is deep enough not to be affected by plants

Permeability and Seepage

Soil Belt: This zone is constantly affected by evaporation and plant transpiration. Moist soil in contact with the atmosphere either evaporates water or condense water into itself until its vapour pressure equal to atmospheric pressure. Soil water in atmospheric equilibrium is called hygroscopic water, whilst the moisture content is known as the hygroscopic water content.

Permeability and Seepage



Permeability and Seepage

Surface Tension: is the property of water that permits the surface molecules to carry a tensile force. Water molecules attract each other and within a mass of water, therefore, balance out. At the surface, however, the molecules only attracted inwards and towards each other which creates surface tension. Surface tension causes the surface of a body of water to attempt to contract into a minimum area, hence a drop of water is spherical.

Permeability and Seepage

Surface tension is given the symbol, T and can be defined as the force in Newton per millimeter length that the water surface can carry. T varies slightly with temperature, but the variation is small and an average value usually taken for the surface tension of water is 0.000075 N/mm or 0.075 N/m .

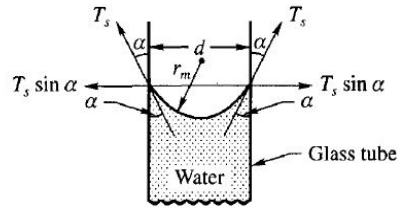
Permeability and Seepage

Capillarity: The fact that surface tension exists can be shown by the familiar laboratory experiment in which an open ended glass capillary tube is placed in a basin of water. Subjected to atmospheric pressure, the rise of water within the tube is then observed. It is seen that the water wets the glass and the column of water within the tube reaches a definite height above the liquid in the basin.

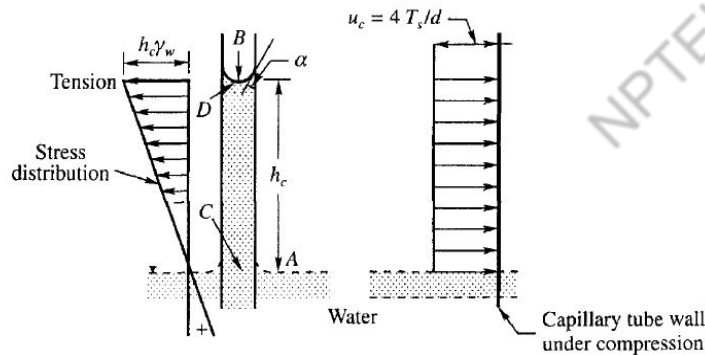
Permeability and Seepage

The surface of the water forms a meniscus such that the curved surface of the liquid is at an angle α , to the walls of the tube. The base of the column is at the same level as this water in the basin and as the system is open, the pressure must be atmospheric. The pressure at the top surface of the column is also atmospheric. There are no externally applied forces that keep the column in position, which shows that there must be a tensile force acting within the surface film of the water

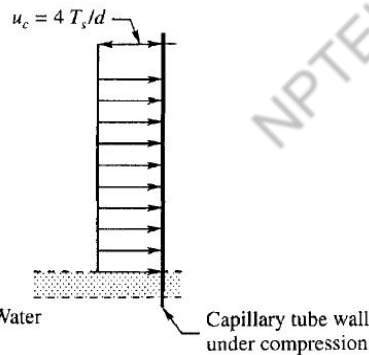
Permeability and Seepage



(a) Forces due to surface tension



(b)



(c)

Permeability and Seepage

h_c is the ht of water column, r is the radius of the tube, γ_w is the unit water of water. Considering force equilibrium in vertical direction:

$$T \times 2\pi r \cos \alpha + \pi r^2 u = 0$$

$$u = \frac{-2T \cos \alpha}{r}$$

$$u = -\gamma_w h_c$$

Permeability and Seepage

Hence as expected, we see that u is negative, the water within the column is in a state of suction. The maximum value of this negative pressure is and occurs at the top of the column.

An expression for height, h_c can be obtained by substituting in the above expression.

$$h_c = \frac{2T \cos \alpha}{r\gamma_w}$$

Permeability and Seepage

From the above two equation we see that the magnitudes of both u and h_c increases as r decreases. With the use of expression for h_c , we can obtain an estimate of the theoretical capillary rise that will occur in a clay deposit. The average void size in clay is about $3\text{ }\mu\text{m}$ and taking $\alpha = 0$, the formula gives, $h_c = 5.0\text{ m}$.

However capillary rise of this magnitude seldom occur in practice as the upward velocity of water flow through a clay in capillary fringe is extremely small and often restricted by adsorbed water films which considerably reduces the free diameter of voids

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SOIL MECHANICS/GEOTECHNICAL ENGINEERING I

SEEPAGE AND PERMEABILITY

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Application: Index properties/classification

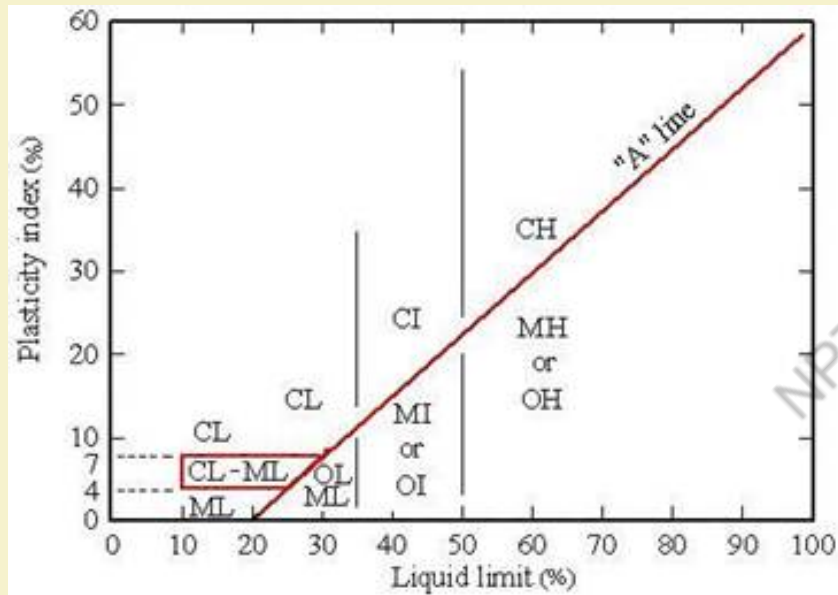
Two clays A and B have the following properties:

Soil properties	Clay A	Clay B
Liquid limit %	44	55
Plastic limit %	20	35
Natural water content%	30	50

Which of the soil is more plastic? Which of them is softer in consistency? Classify the soil as per IS classification system.

Application: Index properties/classification

In a shrinkage limit test, a shrinkage dish of volume 9.66 cc was used. The weight of the saturated soil slurry required to fill the shrinkage dish was 17.5 gm. The slurry was gradually dried first in atmosphere and then in an oven at a constant temperature of 110 deg Celsius. The weight and volume of the dried soil were 11.6 gm and 5.22 cc, respectively. Determine the shrinkage limit of the soil.



Consistency of cohesive soil

Consistency	Description	I_c	I
liquid	liquid	Less than 1	Greater than 1
plastic	Very soft	0-0.25	0.75-1.0
	Soft	0.25-0.5	0.5-0.75
	Medium stiff	0.5-0.75	0.25-0.5
	Stiff	0.75-1.0	0-0.25
Semi solid	V stiff or hard	Greater than 1	Less than 1
solid	Hard/V hard	do	

Seepage and Permeability

Permeability is defined as the property of a soil which allows the seepage of fluids through its interconnected void spaces.

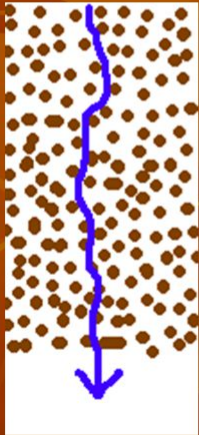
Darcy's Law: $v \propto i$ or $v = ki$

Where k is the coefficient of permeability, v is the discharge velocity and i is the hydraulic gradient.

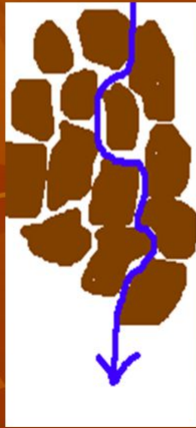
Seepage and Permeability

Soil Structure - Permeability

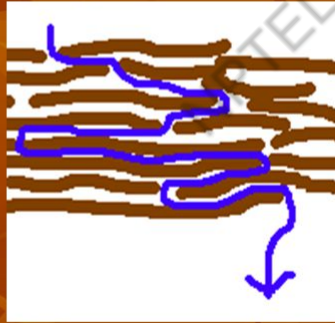
Granular



Blocky



Platy



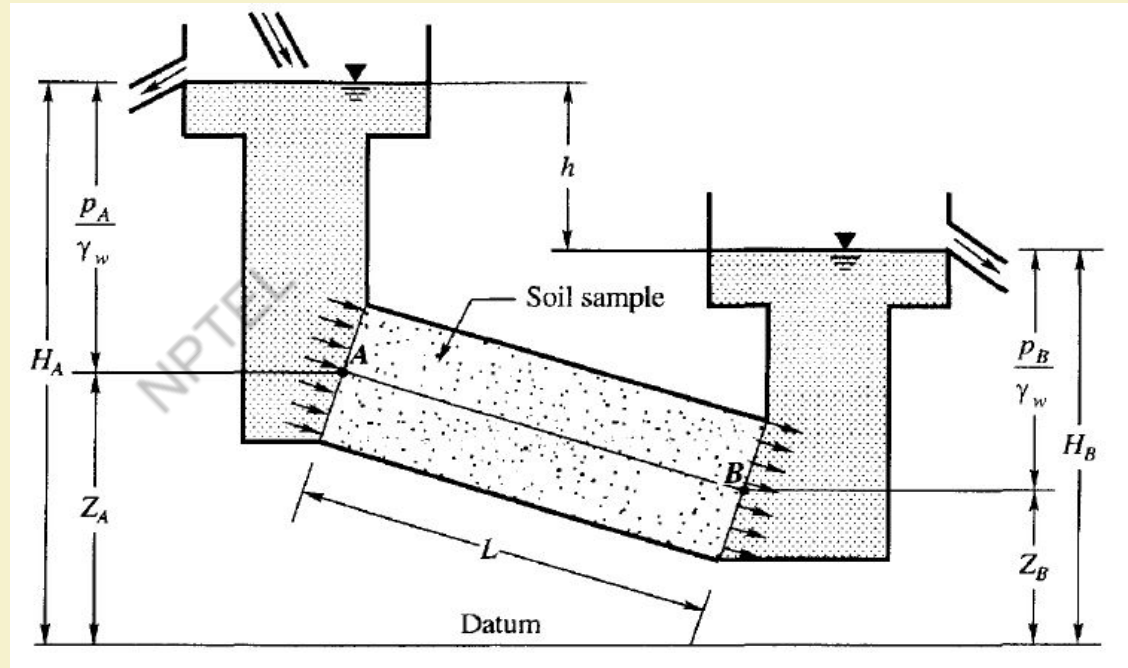
Seepage and Permeability

Total head = elevation head + pressure head + velocity head. Seepage velocity is very less hence velocity head can be neglected

Head at A = $Z_A + h_A$, Head at B = $Z_B + h_B$, Loss of head,
 $\Delta h = (Z_A + h_A) - (Z_B + h_B)$

$$\text{Hydraulic Gradient} = i = \frac{\Delta h}{l}$$

Seepage and Permeability



Seepage and Permeability

- The formula most often used is one produced by Hazen (1892) who stated that for clean sand,

$$k = 10D_{10}^2 \text{ mm/s}$$

- Where k is in mm/s and D_{10} effective size in mm

Seepage and Permeability

Typical values of permeability of different soils

Soil types	Permeability values (m/s)
Gravel	$10^{-1} - 10^{-5}$
Sands	$10^{-5} - 10^{-7}$
Fine sands, coarse silts	$10^{-7} - 10^{-9}$
Silts	$< 10^{-9}$
Clays	

Seepage and Permeability

Factors affecting the coefficient of Permeability:

- shape and size of the soil particles
- void ratio. Permeability increases with increase in void ratio
- degree of saturation. Permeability increases with the increase of degree of saturation
- composition of soil particles; For sand and silts this is not important however for soils with clay minerals this is one of the most important factors. Permeability in this case depends on the thickness of the water held to the soil particles, which is a function of cation exchange capacity, valence of the cation, etc. Other factors remaining the same, the coefficient of permeability decreases with increasing thickness of the diffuse double layer

Seepage and Permeability

Factors : contd...

- Soil structure; Fine grained soils with a flocculated structure have a higher coefficient of permeability than those with dispersed structure. With the increase of moisture content the soil becomes more and more dispersed. With increasing degree of dispersion, the permeability decreases
- Viscosity of permeant
- Density and concentration of permeant

Seepage and Permeability

Determination of Permeability of soil

Laboratory method:

- Falling Head method: good for fine grained soil
- Constant head method: Good for coarse grained soil

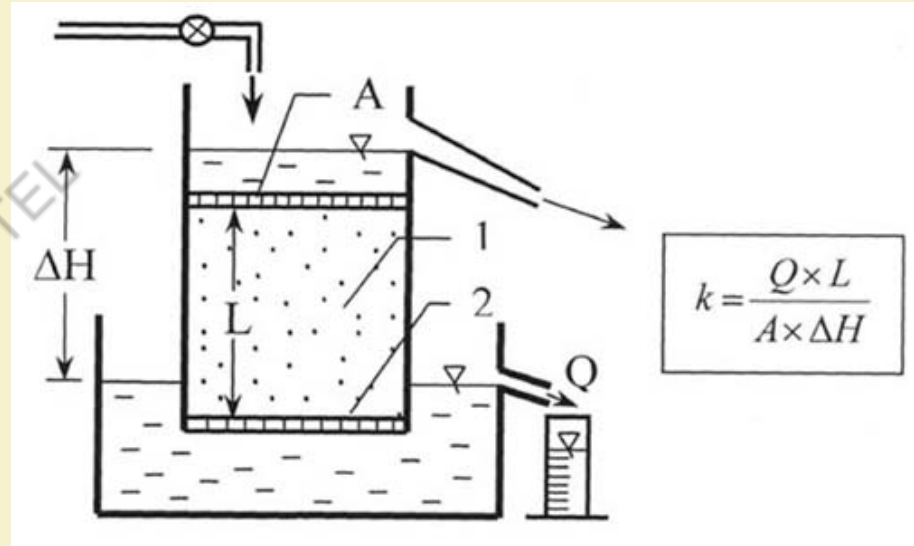
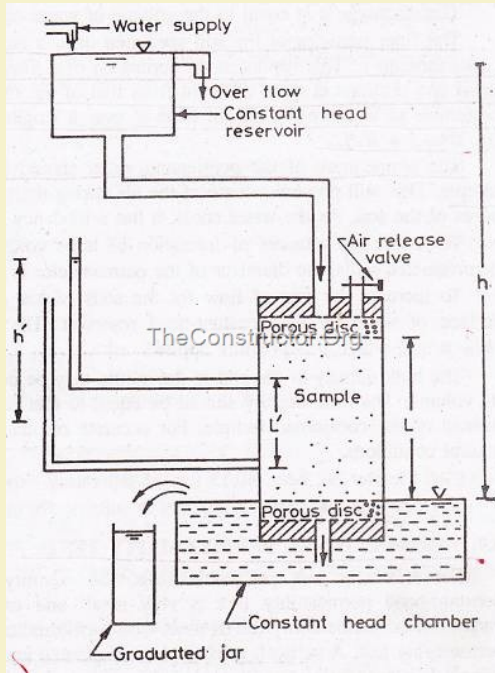
Seepage and Permeability

Field Method:

- Pumping in test
- Pumping out test
- Packer test
- Bore hole test

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Seepage and Permeability



Seepage and Permeability

From Darcy's Law: $q = Aki$

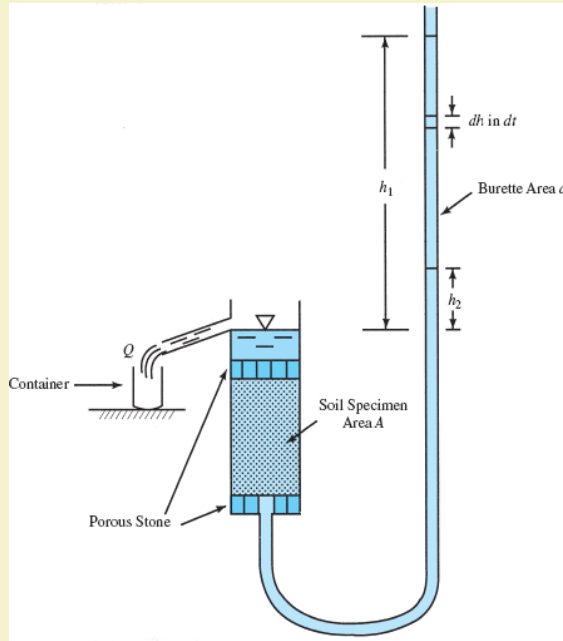
The quantity of Flow, $q = \frac{Q}{t}$

Where i = the hydraulic gradient = h/l , A = cross sectional area of the sample, Q = total quantity of flow over time t

Hence k can be obtained from the expression:

$$k = \frac{q}{Ai} = \frac{Ql}{Ath}$$

Seepage and Permeability



Seepage and Permeability

During the test, the water in the stand pipe falls from a height h_1 to a final height h_2

Let h be the height at some time t . Consider a small time interval dt , and let the change in level of h during this time be $-dh$ (negative as it is a drop in elevation)

The quantity of flow through the sample in time $dt = -a dh$ and is given the symbol, dQ . Now

$$dQ = Ak i dt = Ak \frac{h}{l} dt = -a dh \quad dt = -\frac{al}{Ak} \frac{dh}{h}$$

Seepage and Permeability

$$\int_0^t dt = -\frac{al}{Ak} \int_{h_1}^{h_2} \frac{1}{h} dh$$

$$t = \frac{al}{Ak} \ln \frac{h_1}{h_2} \quad \text{or} \quad k = \frac{al}{At} \ln \frac{h_1}{h_2}$$

A is the cross sectional area of the sample, a is the cross sectional area of the stand pipe, l is the length of the sample, t is the time over which head dropped from h_1 to h_2

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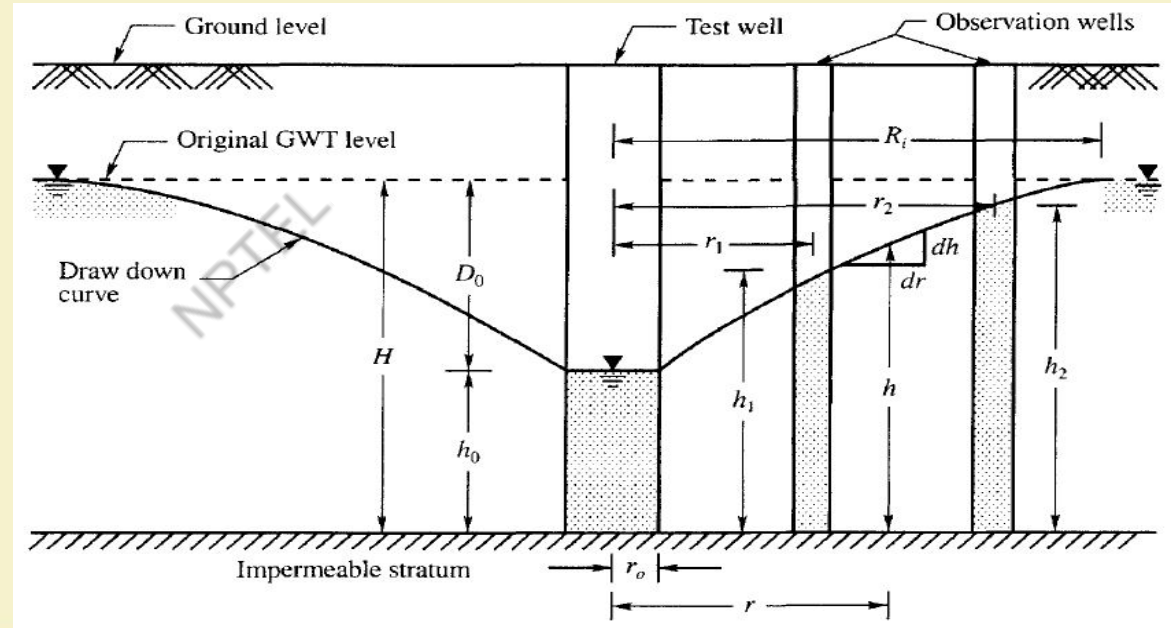
PERMEABILITY AND SEEPAGE

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Permeability and Seepage

✓ Field Permeability test: Pumping test

Unconfined aquifer



Permeability and Seepage

Consider an intermediate distance r from the centre line of the pumping well and let the height of GWL above the impermeable layer during pumping be h

The hydraulic gradient, i , is equal to the slope of the $h - r$ curve $= \frac{\partial h}{\partial r}$

Area of imaginary wall of the cylinder of radius r and height $h = 2\pi r h$

$$q = Aki = 2\pi r h k \frac{\partial h}{\partial r} \quad \text{or} \quad q \frac{\partial r}{r} = 2\pi k h \partial h$$

Permeability and Seepage

$$q \int_{r_1}^{r_2} \frac{1}{r} dr = 2\pi k \int_{h_1}^{h_2} h dh \quad \Rightarrow \quad k = \frac{q \ln\left(\frac{r_2}{r_1}\right)}{\pi(h_2^2 - h_1^2)}$$

Permeability and Seepage

According to Kozeny the maximum radius of influence, R for drawdown due to pumping is given by,

$$R = \sqrt{\frac{12t}{n} \sqrt{\frac{qk}{\pi}}}$$

Where n = porosity, R = radius of influence, and t = time during which discharge of water from well has been established

Also if $h_1 = h_0, r_1 = r_w$, and $h_2 = H$ at $r_2 = R$ are substituted

$$k = \frac{q \ln(R/r_0)}{\pi(H^2 - h_0^2)}$$

Permeability and Seepage

The depth h at any distance r from the well $r_w \leq r \leq R$ can be determined from the previous equation derived by substituting $h_1 = h_0$ at $r_1 = r_w$ and $h_2 = h$ at $r_2 = r$

$$k = \frac{q \ln(r/r_w)}{\pi(h^2 - h_0^2)}$$



$$h = \sqrt{\frac{q}{\pi k} \ln(r/r_w) + h_0^2}$$

Permeability and Seepage

Area of imaginary wall of the cylinder of radius r and height $h = 2\pi r H$

$$q = Aki = 2\pi r H k \frac{\partial h}{\partial r} \quad \text{or} \quad q \frac{\partial r}{r} = 2\pi k H \partial h$$

$$q \int_{r_1}^{r_2} \frac{1}{r} \partial r = 2\pi k H \int_{h_1}^{h_2} \partial h \quad \longrightarrow \quad k = \frac{q \ln\left(\frac{r_2}{r_1}\right)}{2\pi H (h_2 - h_1)}$$

Permeability and Seepage

A sedimentary deposit may consist of several different soils and it is often necessary to determine the average values of permeability in two directions one parallel to bedding planes and the other at right angles to them

Let there be n layers of thicknesses, $H_1, H_2, H_3, \dots, H_n$ and total thickness of the layers be H

Let $k_{x1}, k_{x2}, k_{x3}, \dots, k_{xn}$ be the representative coefficient of permeability parallel to bedding plane and k_{xe} be the average permeability parallel to bedding plane

Considering flow parallel to bedding plane $q = Ak_{xe}i$; where A is the total area and i is the hydraulic gradient

Permeability and Seepage

The total flow must equal the sum of the flow through each layer therefore:

$$Ak_{xe}i = A_1k_{x1}i + A_2k_{x2}i + A_3k_{x3}i + \dots \dots \dots + A_nk_{xn}i$$

Considering unit width, i.e, $A=H$ and hence,

$$Hk_{xe}i = i(H_1k_{x1} + H_2k_{x2} + H_3k_{x3} + \dots \dots + H_nk_{xn})$$



$$k_{xe} = \frac{H_1k_{x1} + H_2k_{x2} + H_3k_{x3} + \dots \dots + H_nk_{xn}}{H_1 + H_2 + H_3 + \dots \dots + H_n} = \frac{\sum_1^n H_n k_n}{\sum_1^n H_n}$$

Permeability and Seepage

Considering flow perpendicular to the plane and considering $k_{z1}, k_{z2}, k_{z3}, \dots \dots \dots k_{zn}$ representative permeability perpendicular to the bedding plane in the respective layers

Total flow, $q = Ak_{z1}i_1 = Ak_{z2}i_2 = Ak_{z3}i_3 = \dots \dots \dots = Ak_{zn}i_n$

$q = k_{ze}i = k_{z1}i_1 = k_{z2}i_2 = k_{z3}i_3 = \dots \dots = k_{zn}i_n$ Considering unit area

If $h_1, h_2, h_3, \dots \dots \dots h_n$ head losses across each layers

$$k_{ze}i = k_{ze} \frac{h_1 + h_2 + h_3 + \dots \dots + h_n}{H}$$

Permeability and Seepage

$$q = \frac{k_{z1} h_1}{H_1}; q = \frac{k_{z2} h_2}{H_2}; q = \frac{k_{z3} h_3}{H_3}; \dots q = \frac{k_{zn} h_n}{H_n}$$

$$h_1 = \frac{q H_1}{k_{z1}}; h_2 = \frac{q H_2}{k_{z2}}; h_3 = \frac{q H_3}{k_{z3}}; h_n = \frac{q H_n}{k_{zn}}$$

$$\frac{k_{ze} \left(\frac{q H_1}{k_{z1}} + \frac{q H_2}{k_{z2}} + \frac{q H_3}{k_{z3}} + \dots + \frac{q H_n}{k_{zn}} \right)}{H} = q$$

$$k_{ze} = \frac{H}{\frac{H_1}{k_{z1}} + \frac{H_2}{k_{z2}} + \frac{H_3}{k_{z3}}} \quad \longrightarrow \quad k_{ze} = \frac{H}{\sum_1^n \frac{H_n}{k_n}}$$

Thank You!!

