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SOIL MECHANICS/GEOTECHNICAL ENGINEERING I

VERTICAL STRESS DISTRIBUTION OVER DEPTH

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Stresses in Soils

Within the context of geotechnical engineering analysis it is convenient to view the in-situ soil stress, at a given point, in terms of the components of total stress:

1. Stress induced by the weight of the soil above that level
2. Fluid pressure
3. Stresses introduced by externally applied loads (if any)

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VERTICAL STRESS DISTRIBUTION

Boussinesq's theory: He has given a solution in (1885)for finding out the stress components at a depth, z, below the surface of a semi-infinite solid due to a pint load acting at the surface of the semi-infinite soil.

Assumptions:

- The soil mass is elastic, isotropic, homogeneous and semi-infinite
- The soil is weightless
- The load is applied at a point on the surface

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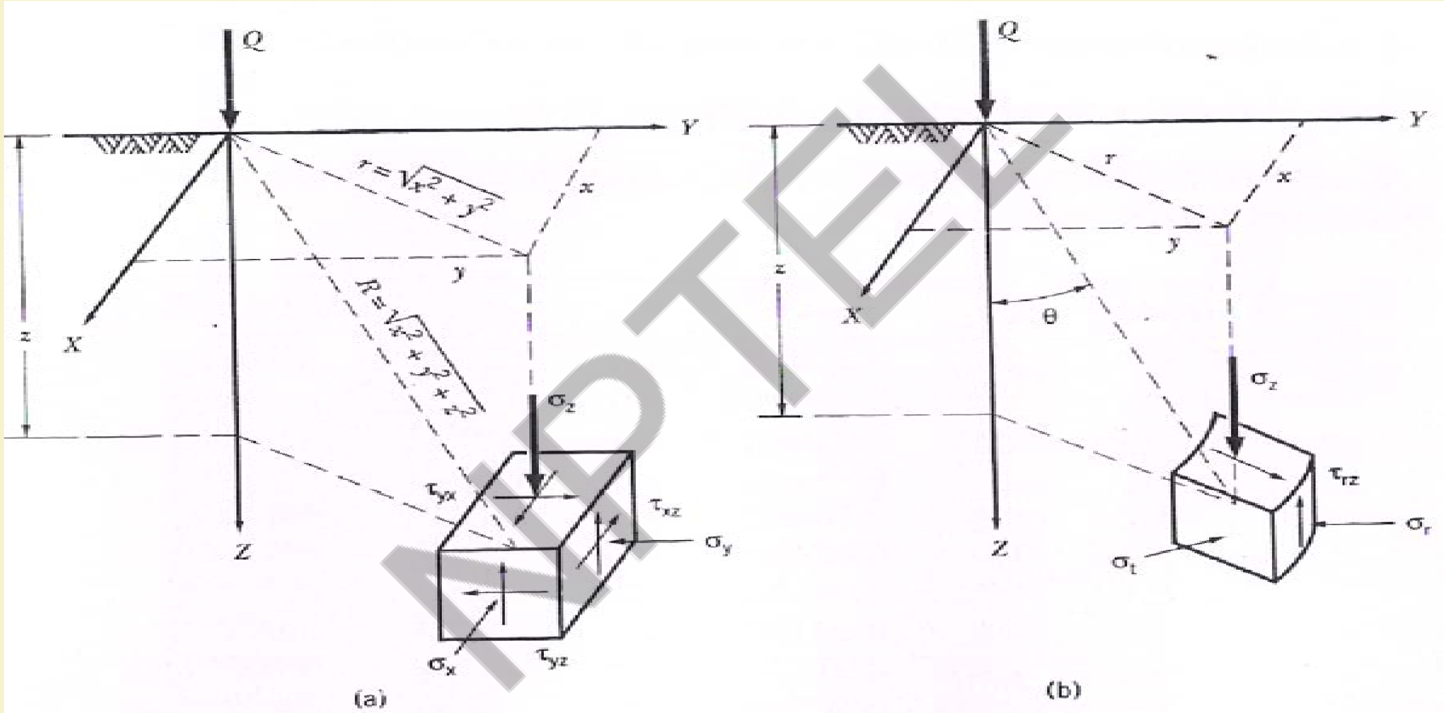

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VERTICAL STRESS DISTRIBUTION

In rectangular coordinates

$$\sigma_z = \frac{3Q}{2\pi}\frac{z^3}{R^5}$$

$$\sigma_x = \frac{3Q}{2\pi}\left\{\frac{x^2 z}{R^5} + \frac{1-2\mu}{3}\left[\frac{1}{R(R+z)} - \frac{(2R+z)\,x^2}{R^3(R+z)^2} - \frac{z}{R^3}\right]\right\}$$

$$\sigma_y = \frac{3Q}{2\pi}\left\{\frac{y^2 z}{R^5} + \frac{1-2\mu}{3}\left[\frac{1}{R(R+z)} - \frac{(2R+z)\,y^2}{R^3(R+z)^2} - \frac{z}{R^3}\right]\right\}$$

VERTICAL STRESS DISTRIBUTION

$$\tau_{zx} = -\frac{3Q}{2\pi}\frac{xz^2}{R^5}$$

$$\tau_{xy} = \frac{3Q}{2\pi}\left[\frac{xyz}{R^5} - \frac{1-2\mu}{3}\frac{(2R+z)\,xy}{R^3(R+z)^2}\right]$$

$$\tau_{yz} = -\frac{3Q}{2\pi}\frac{yz^2}{R^5}$$

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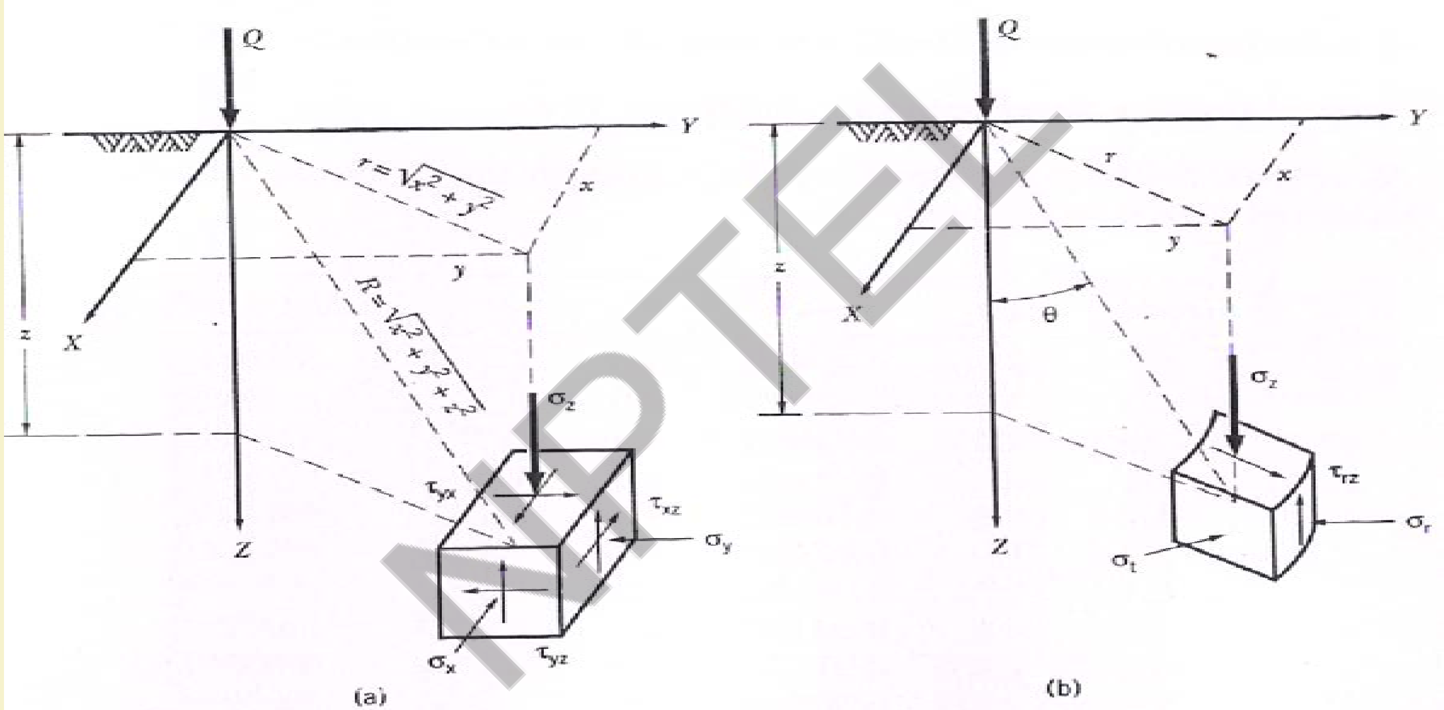

VERTICAL STRESS DISTRIBUTION

In polar coordinates

$$\sigma_z = \frac{Q}{2\pi}\frac{3z^3}{(r^2+z^2)^{5/2}} = \frac{Q}{2\pi z^2}(3\cos^5\theta)$$

$$\sigma_r = \frac{Q}{2\pi}\left[\frac{3r^2z}{(r^2+z^2)^{5/2}} - \frac{1-2\mu}{r^2+z^2+z\sqrt{r^2+z^2}}\right]$$

$$= \frac{Q}{2\pi z^2}\left[3\sin^2\theta\cos^3\theta - \frac{(1-2\mu)\cos^2\theta}{1+\cos\theta}\right]$$

$$\sigma_t = -\frac{Q}{2\pi}(1-2\mu)\left[\frac{z}{(r^2+z^2)^{3/2}} - \frac{1}{r^2+z^2+z\sqrt{r^2+z^2}}\right]$$

$$= -\frac{Q}{2\pi z^2}(1-2\mu)\left[\cos^3\theta - \frac{\cos^2\theta}{1+\cos\theta}\right]$$

$$\tau = \frac{Q}{2\pi}\frac{3rz^3}{(r^2+z^2)^{5/2}} = \frac{Q}{2\pi z^2}(3\sin\theta\cos^4\theta)$$

VERTICAL STRESS DISTRIBUTION

$$\sigma_z = \Delta\sigma = \frac{3Q}{2\pi z^2}\frac{1}{\left[1+\frac{r^2}{z^2}\right]^{5/2}}$$

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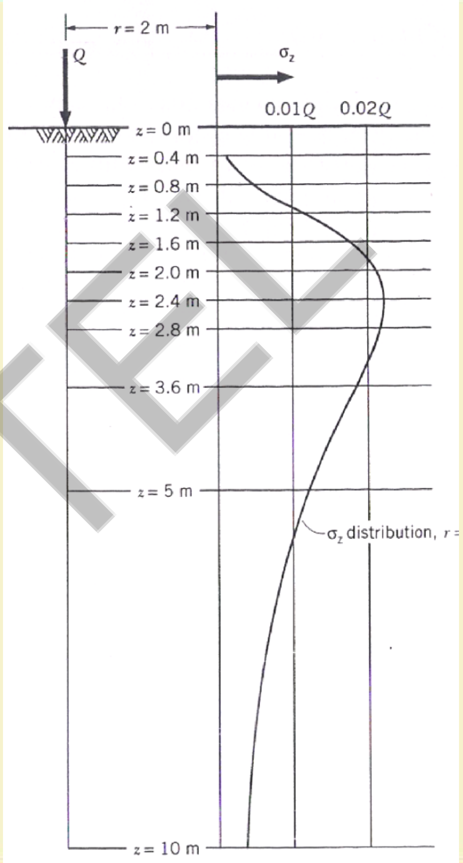

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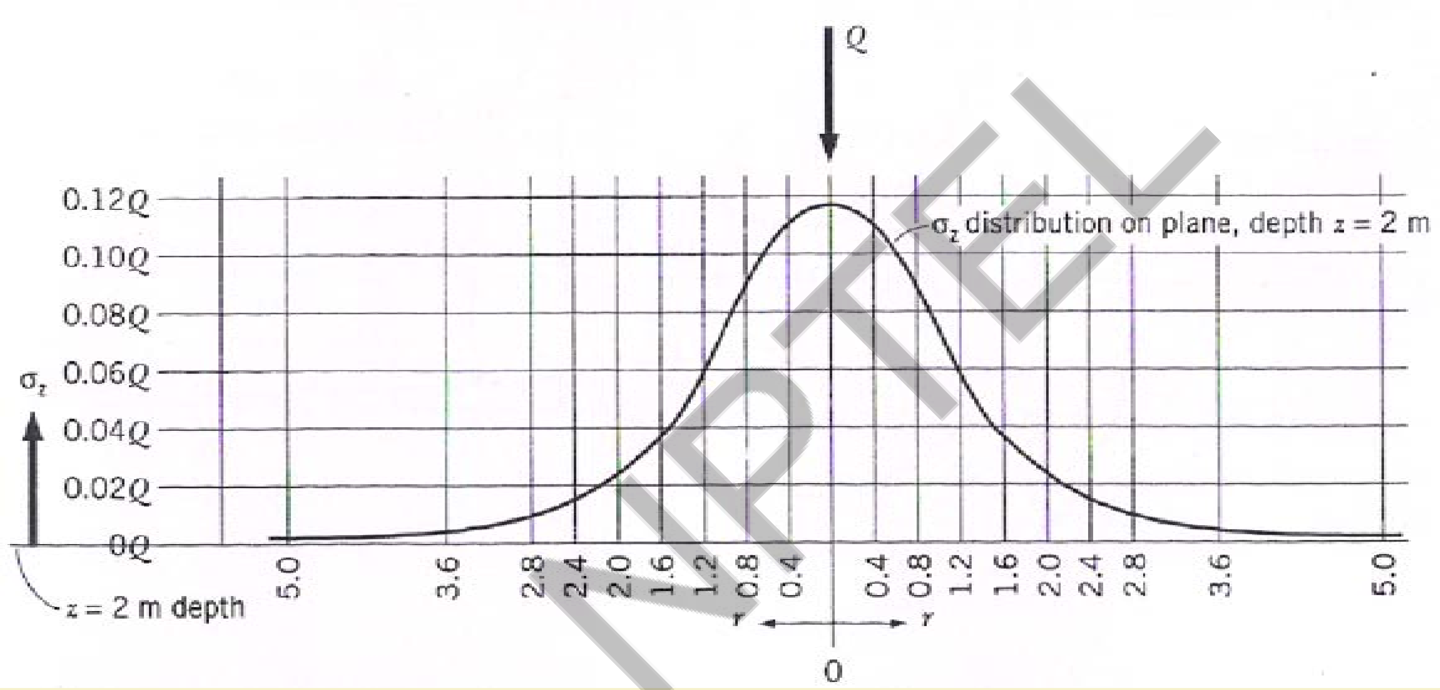

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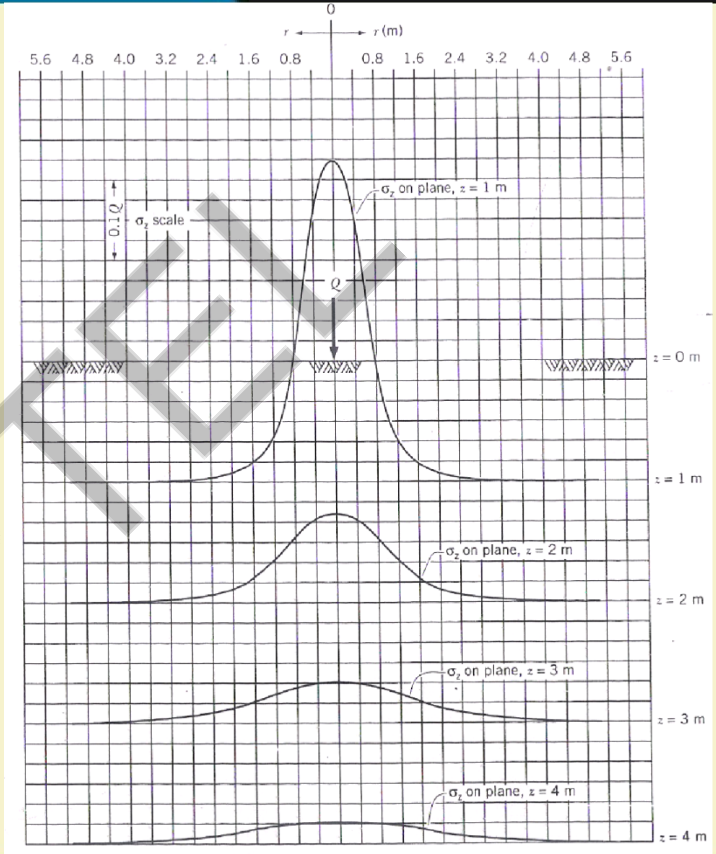

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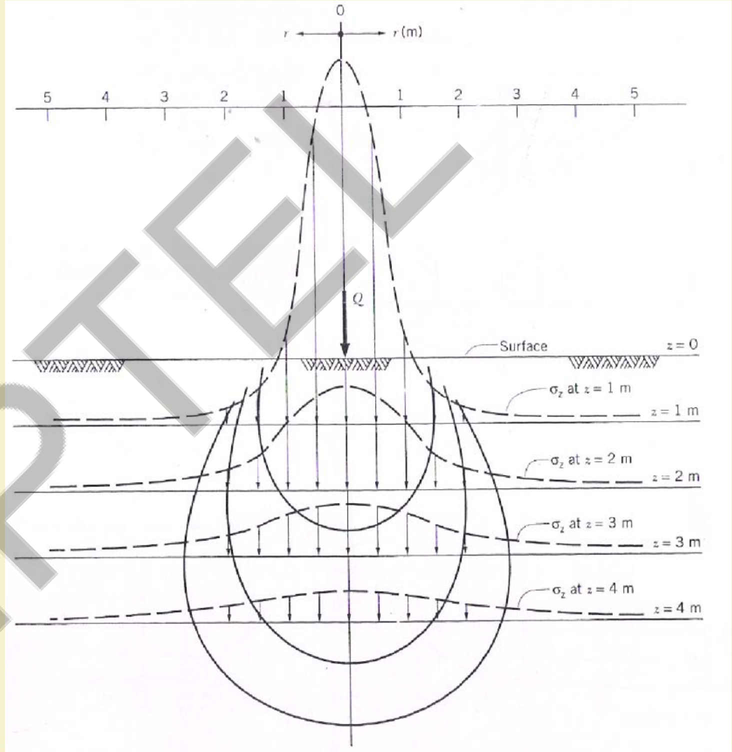

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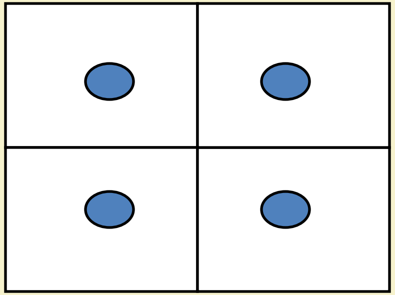


Application of Boussinesq's point load formula: For a square footing of size 3 m by 3 m subjected to a pressure of 100 kPa. For this loading, find out vertical stress at any depth, (at 2 m depth) below the center of the footing.

Total load on the footing is 9x100 = 900 kN, considering a point load acting through the center of the footing and substituting z = 2 m, r =0.0 into the equation:

$$\sigma_z = \Delta\sigma = \frac{3Q}{2\pi z^2}\frac{1}{\left[1+\frac{r^2}{z^2}\right]^{5/2}}$$

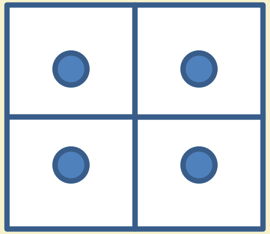
Reduces to : $\sigma_z = \Delta\sigma = \frac{3Q}{2\pi z^2} = 107.43\, kN/m^2$

This is highly an approximation

VERTICAL STRESS DISTRIBUTION

More accurate value can be obtained using the same Boussinesq formula but with suitable modification. The entire foundation area can be divided into a number of small units and Boussinesq formula can applied on the small unit and finally cumulative effect of all the parts at any desired location can be obtained by summing them. Smaller the unit size better is the accuracy. The same problem can be solved by diving the foundation area into 4 small parts and 9 small parts and its effect on the final results can be seen.

VERTICAL STRESS DISTRIBUTION

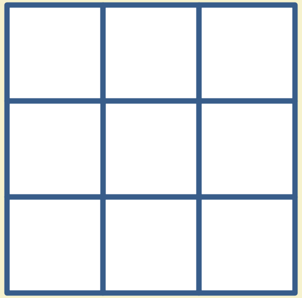


Load in each unit = Q1 =Q2 = Q3 = Q4 =100x1.5x1.5 = 2.25 kN . At the same depth of 2 m below the center of the foundation is desired. Now Q1 can be applied in any part and stress can be calculated below the center of the footing. Distance between the center of the footing to the point of load application is r here. Here r1 = r2 = r3 = r4 for all four parts = 1.06 m.

$$\Delta p_1 = \frac{3Q_1}{2\pi z^2}\frac{1}{\left[1+{r^2}/{z^2}\right]^{5/2}} = \frac{3\times 2.25}{2\pi\times 2^2}\frac{1}{\left[1+{1.06^2}/{2^2}\right]^{2.5}} = 14.46\ kN/m^2$$

$$\Delta p = 4\times\Delta p_1 = 4\times 14.46 = 57.85\ kN/m^2$$

VERTICAL STRESS DISTRIBUTION



$R1 = 0.0$, $r2 = 1$ m, $r3 = \sqrt{(2)}$

$\Delta p1 = 11.936$
$\Delta p2 = 6.833$
$\Delta p3 = 4.331$
$\Delta p = \Delta p1 + 4\,\Delta p2 + 4\,\Delta p3 = 56.592$ kN/m2

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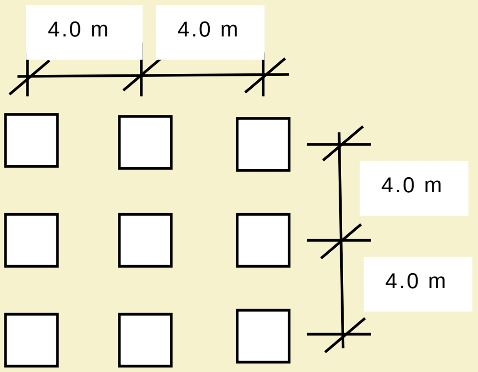
VERTICAL STRESS DISTRIBUTION

A system of isolated footings for a building block is shown in Figure . The size of each footing is 1.8 m × 1.8 m and each footing exerts a pressure of 287 kPa on the underlying soil. Determine the vertical pressure at a depth of 3 m below the center of footing 1, 2, and 5 as shown in the figure. The pressure exerted by each footing can be assumed as a single equivalent load acting through its center



Figure

VERTICAL STRESS DISTRIBUTION

Calculation for below footing No 5 is shown here.
Pressure under footing will come from all 8 surrounding footing and the self.
Because of symmetry 3 calculations are required. 4 footings are at a distance r1 = 4m and other four footings are at a distance of 5.66 m from the central footing. For the central footing r = 0.0

$$\Delta p_1 = \frac{3 \times 930}{2 \times \pi \times 3^2} \frac{1}{\left(1 + \frac{4^2}{3^2}\right)^{2.5}} = 3.836 \quad \Delta p_2 = \frac{3 \times 930}{2 \times \pi \times 3^2} \frac{1}{\left(1 + \frac{5.66^2}{3^2}\right)^{2.5}} = 1.111$$

$$\Delta p_0 = \frac{3 \times 930}{2 \times \pi \times 3^2} = 49.338 \qquad \Delta p = \Delta p_0 + 4\Delta p_1 + 4\Delta p_2 = 69.13$$

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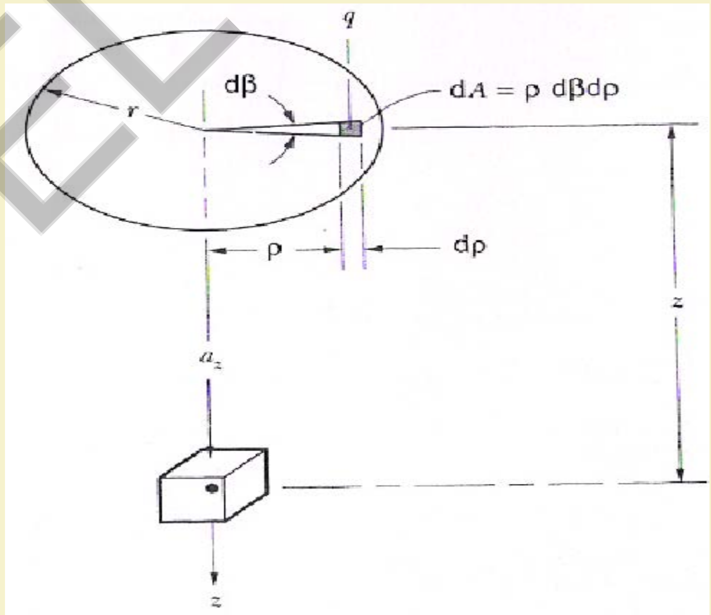
VERTICAL STRESS

The unit vertical stress on any given depth could be determined with acceptable accuracy by extending Boussinesq's equation to a uniformly loaded circular area.

Vertical stress under the centre of the circular footing:



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From Boussinesq's equation

$$\sigma_z = \frac{3q\,z^3}{2\pi}\int_0^{2\pi}\int_0^{r}\frac{\rho\,d\beta\,d\rho}{(\rho^2+z^2)^{5/2}}$$

Integrating with respect to beta and substituting limits, we have

$$\sigma_z = 3qz^3\int_0^{r}\frac{\rho\,d\rho}{(\rho^2+z^2)^{5/2}}$$

Integrating

$$\sigma_z = q\left[1-\frac{1}{\left(\frac{r^2}{z^2}+1\right)^{3/2}}\right]$$

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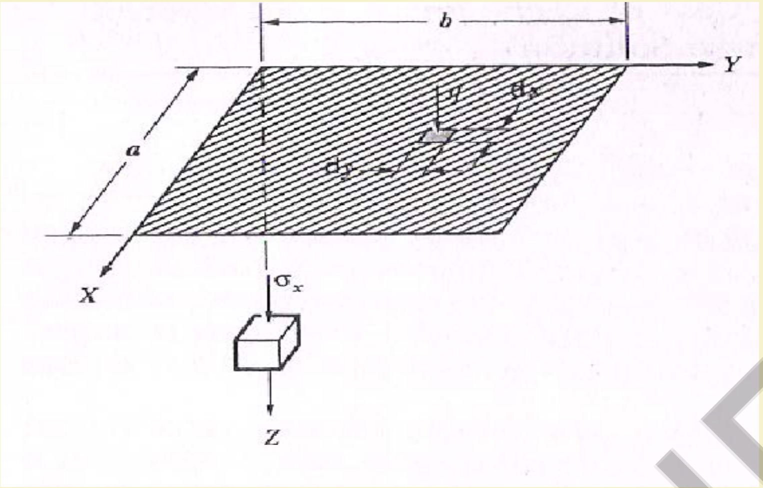

Rectangular area: Corner formula

$$\sigma_z = \frac{3qz^3}{2\pi}\int_0^a\int_0^b \frac{dy\,dx}{(x^2+y^2+z^2)^{5/2}}$$

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VERTICAL STRESS

$$\sigma_z = \frac{q}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{(m^2+n^2+1+mn)}\frac{(m^2+n^2+2)}{(m^2+n^2+1)} + \sin^{-1}\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+1+m^2n^2}\right]$$

Where m =a/z and n = b/z

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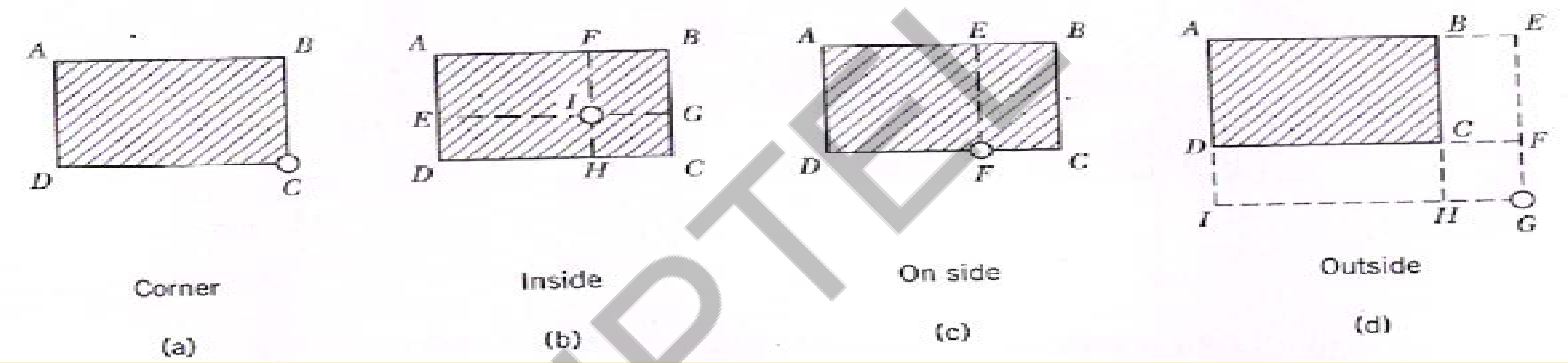

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VERITICAL STRESS

Newmarks's procedure evolves from the expression for the vertical stress under the centre of loaded circular area given by:

$$\sigma_z = q\left[1 - \frac{1}{\left(\frac{r^2}{z^2}+1\right)^{3/2}}\right] \; or \; \frac{\sigma_z}{q} = \left[1 - \frac{1}{\left(\frac{r^2}{z^2}+1\right)^{3/2}}\right]$$

$\frac{\sigma_z}{q}$ Versus r/z

$\frac{\sigma_z}{Q}$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
r/z	0	0.27	0.40	0.52	0.64	0.77	0.92	1.11	1.39	1.91	

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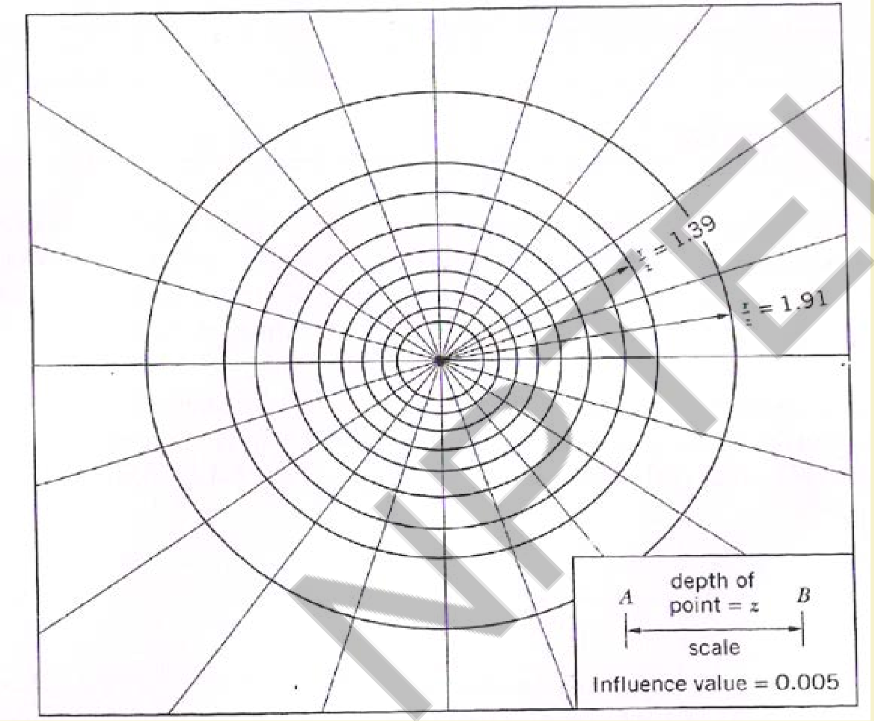
VERITICAL STRESS



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VERITICAL STRESS

Method for drawing Newmarks Chart:

Draw a line of any length representing the depth

$$\Delta\sigma = I\,N\,q$$

Find out r for different $\frac{\sigma_z}{q}$ ratios, say r1, r2, r3……..r11,

Draw circles with different radius so obtained

Draw radial lines dividing the whole circle into twenty equal part

Thus whole area is divided by 10x20= 200 div and Influence value of each small block is 0.005

Now length of the line so chosen for drawing the diagram taken as the desired depth at which stress to be calculated and Draw the whole area in that scale

Cover the chart with the scaled drawn area keeping the point of interest on the centre of the diagram and then count the number of blocks covered by the diagram

Finally stress at that depth is given by,

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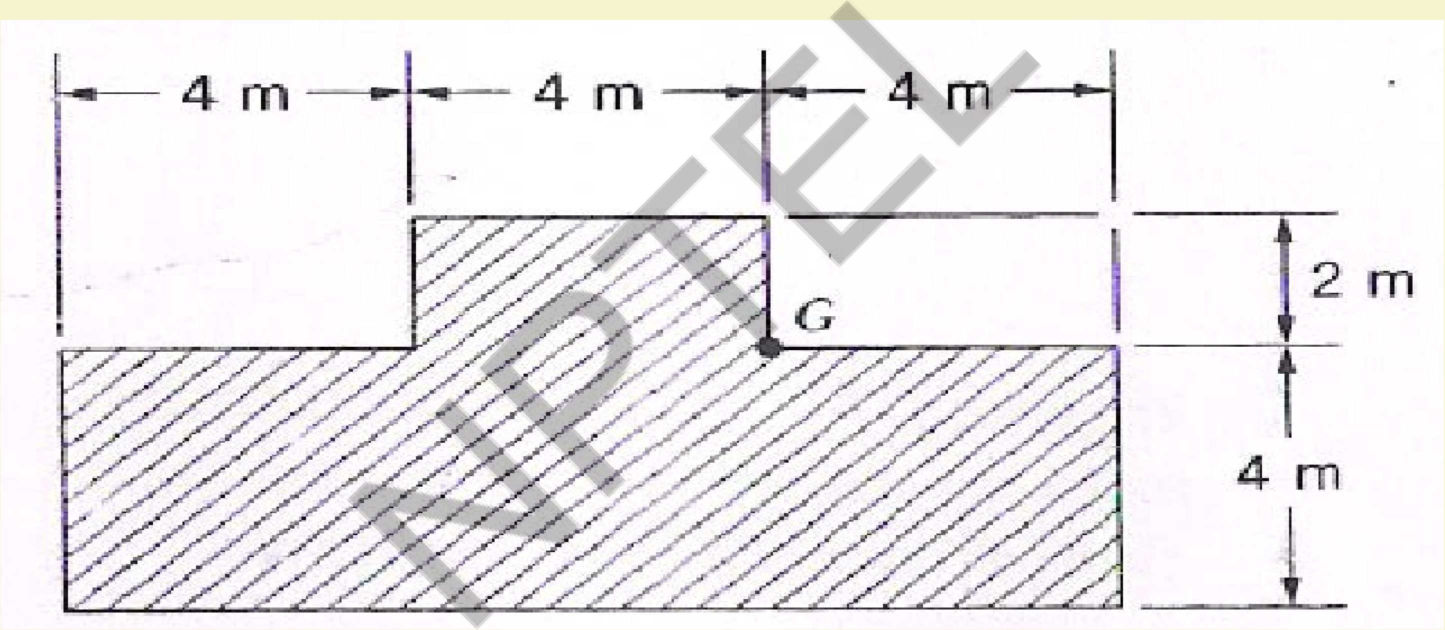

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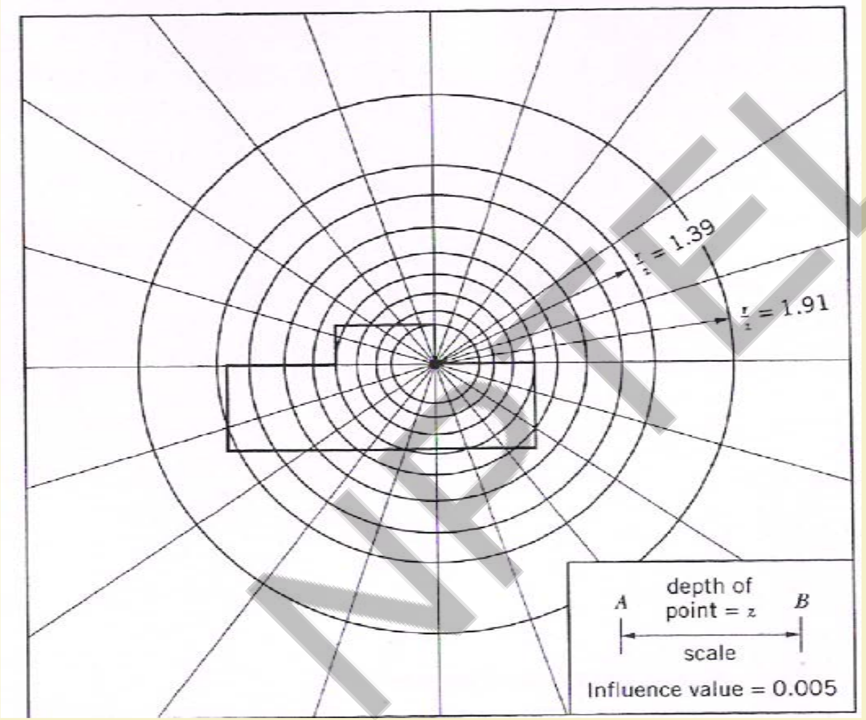

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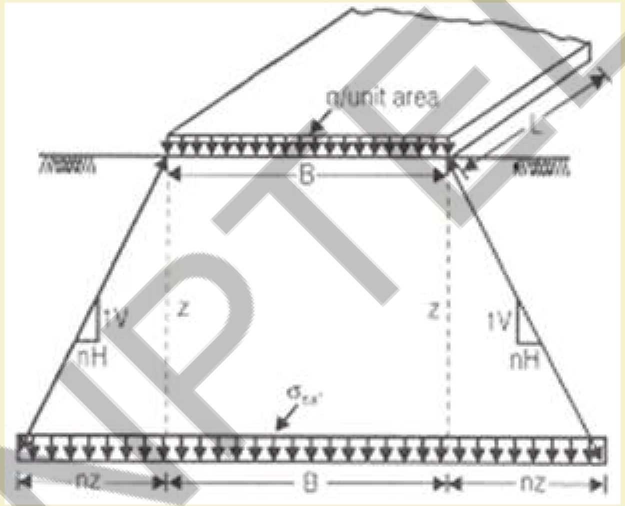
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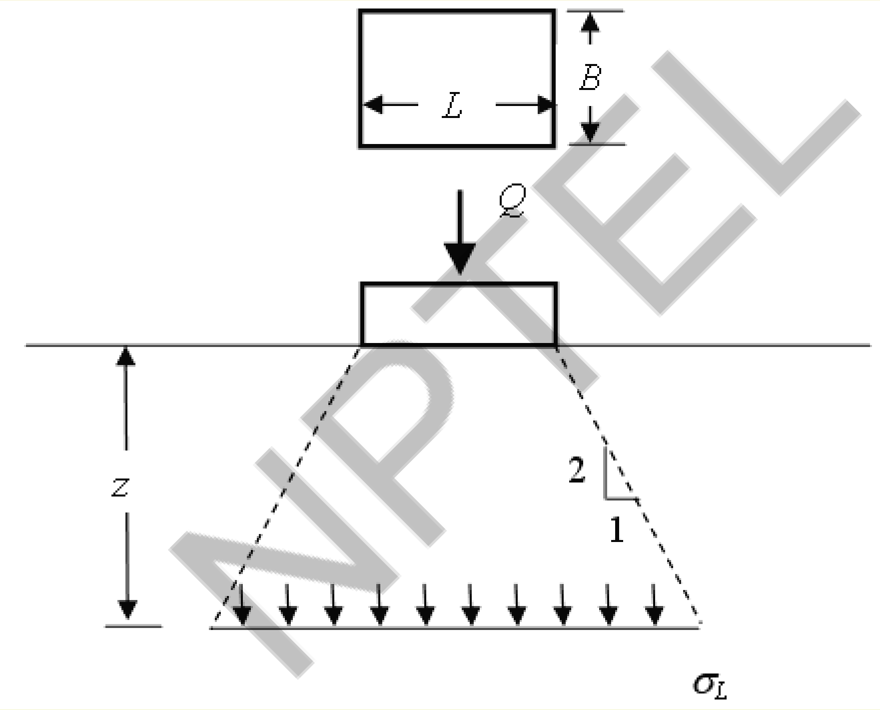
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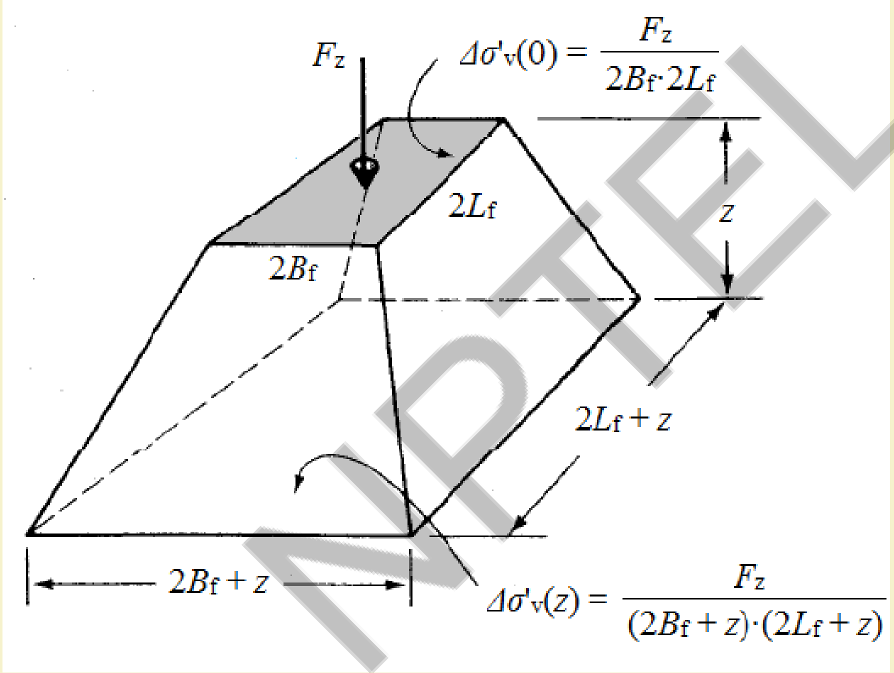
VERITICAL STRESS



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VERTICAL STRESS DISTRIBUTION: APPLICATION

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VERTICAL STRESS: APPLICATION

The base of a tower consists of an equilateral triangular frame of 6.0 m each, on the each corner of which the column of the tower is supported. The total weight of the tower is 600 kN, which is equally shared by all the three columns. Compute the increase in vertical stress in soil caused by the loads of tower at a point 5 m below any one of the columns.

VERTICAL STRESS: APPLICATION

Nine square footings of equal size (1m×1m) arranged in square pattern (3 in each row and 3 in each column). Each footing carries 60 kN load and center to spacing of the footings is 1.8 m. Determine the vertical stress at a depth 5 m below the centre of one of the corner footing.

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VERTICAL STRESS: APPLICATION

An annular footing with inner and outer radius 6 m and 8, respectively is supporting a tank which exerted a pressure intensity of 200 kPa. Determine the pressure increase at a depth of 5 m below the center of the footing because of the foundation pressure.

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