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NPTEL ONLINE CERTIFICATION COURSES

# SOIL MECHANICS/GEOTECHNICAL ENGINEERING I

## SHEAR STRNGTH

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# SHEAR STRENGTH:summary

The shear strength of the soil may be attributed to three basic components:

- Frictional resistance to sliding between solid particles
- Cohesion and adhesion between the soil particles
- Interlocking and bridging of solid particles to resist deformation




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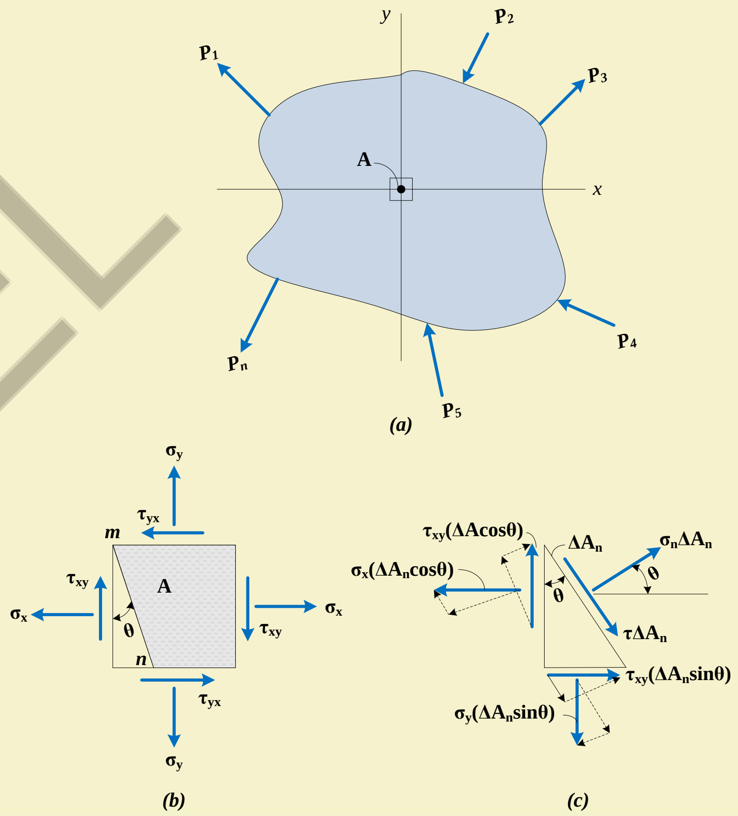
# SHEAR STRENGTH: summary

$$\sigma_n = \frac{(\sigma_x + \sigma_y)}{2} + \frac{(\sigma_x - \sigma_y)}{2} cos2\theta - 2\tau_{xy} sin2\theta$$

$$\tau_n = \frac{(\sigma_x - \sigma_y)}{2} sin2\theta + \tau_{xy} cos2\theta$$

$$tan2\theta = \frac{-\tau_{xy}}{(\sigma_x - \sigma_2)/2}$$



# SHEAR STRENGTH: summary

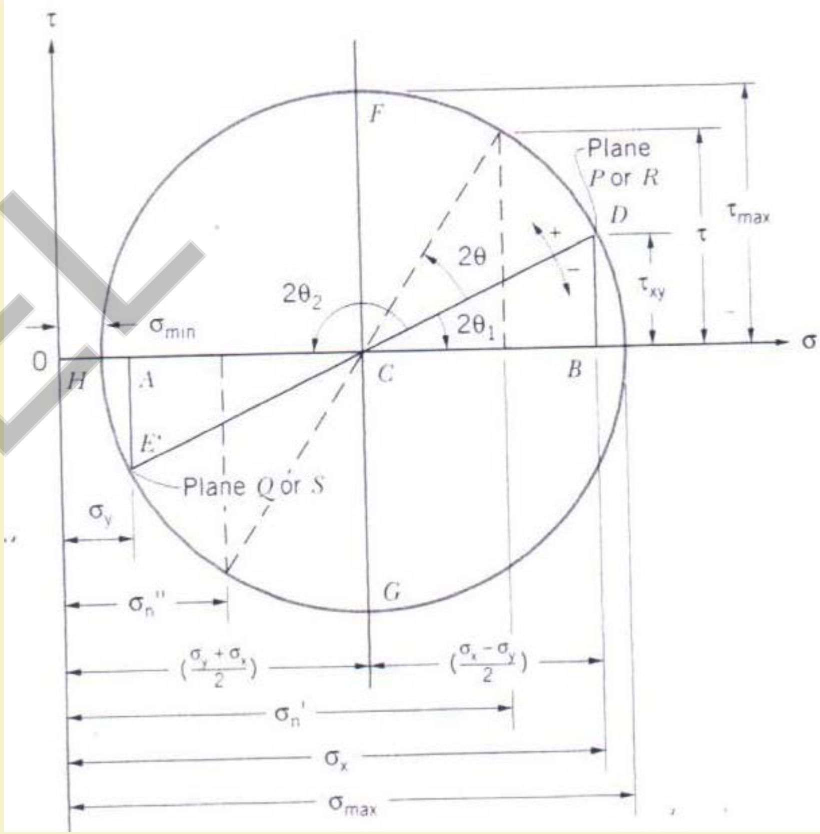
$$(\sigma_n - \frac{\sigma_x + \sigma_y}{2}) + \tau_n^{\,2} = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\,2}$$

The expression is an equation of circle with center at $(\sigma_x + \sigma_y)/2,\ 0$ and radius equal to

$$\sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{xy}^{\,2}}$$

# SHEAR STRENGTH: summary

The positive angles on the circle are obtained when measured in the counterclockwise sense; negative on the circle are obtained in the clockwise sense. An angle of 2θ on the circle corresponds to an angle θ on the element



# SHEAR STRENGTH: summary

**Principal Plane**: A plane that is acted upon by a normal stress only is known as a principal plane, there is no tangential or shear stress present.

**Principal Stress**: The normal stress acting on principal plane is referred to as a principal stress. At every point in a soil mass, applied stress system that exists can be resolved into three principal stresses that are mutually orthogonal. The principal planes corresponding to these principal stresses are called major, intermediate and minor principal planes and are so named from the consideration of the principal stresses that act upon them. The principal stress $\sigma$ is known as the major principal stress acts on the major principal plane.





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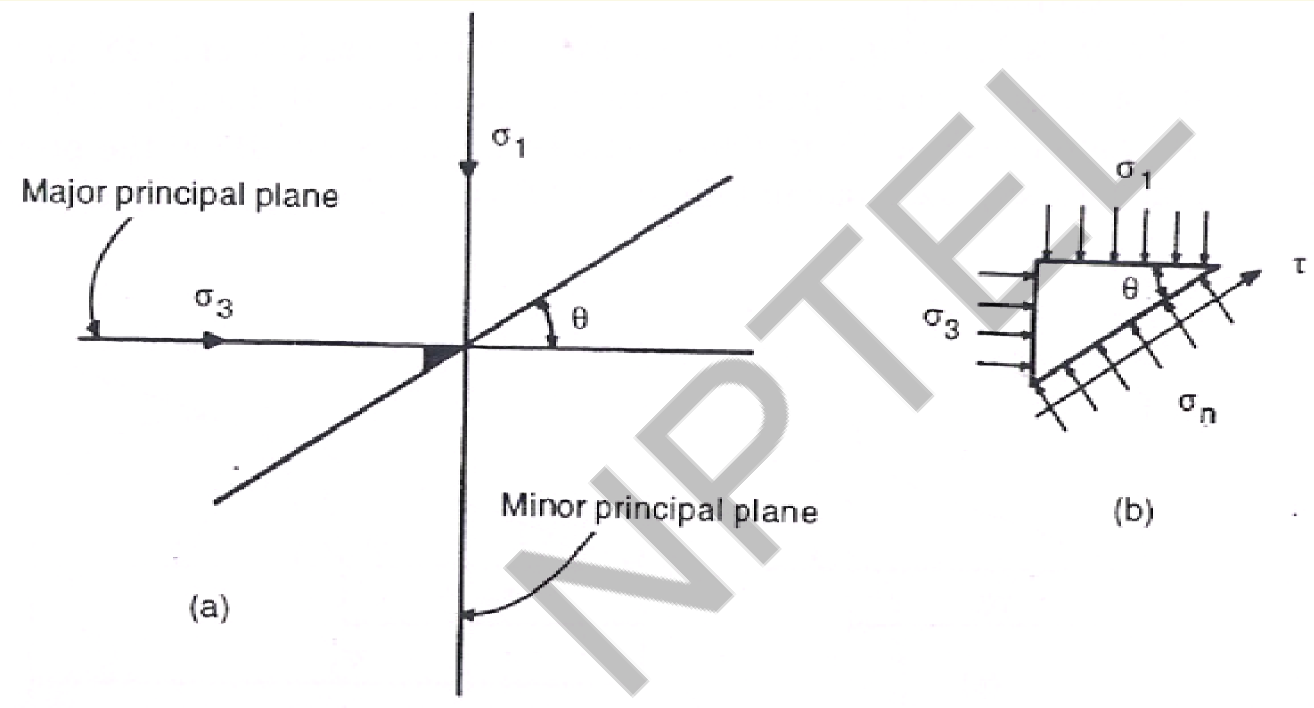
# SHEAR STRENGTH: summary





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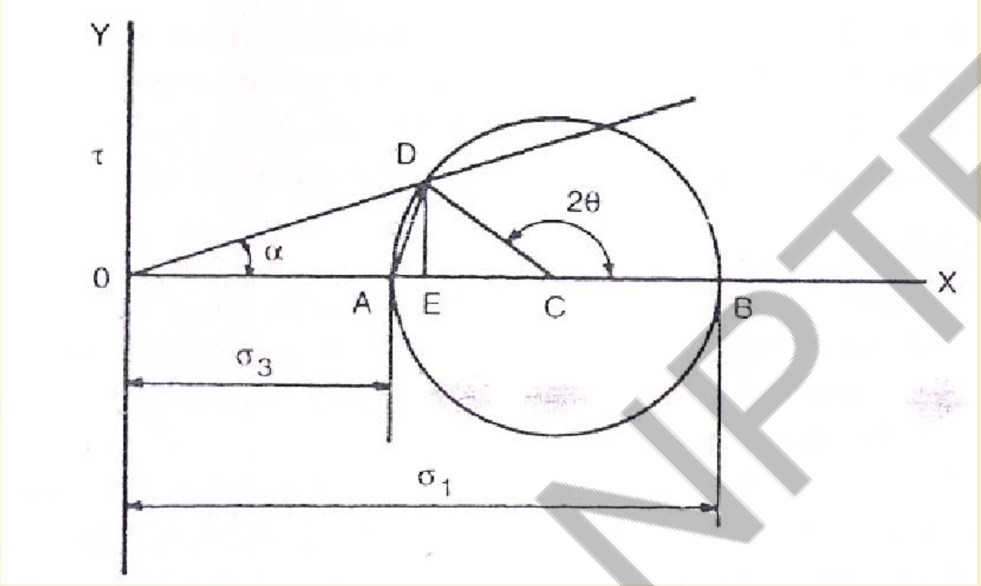
# SHEAR STRENGTH: summary




$$\tau = \frac{\sigma_1 - \sigma_3}{2}\, sin2\theta$$

$$\sigma_n = \sigma_3 + (\sigma_1 - \sigma_3)cos^2\theta$$

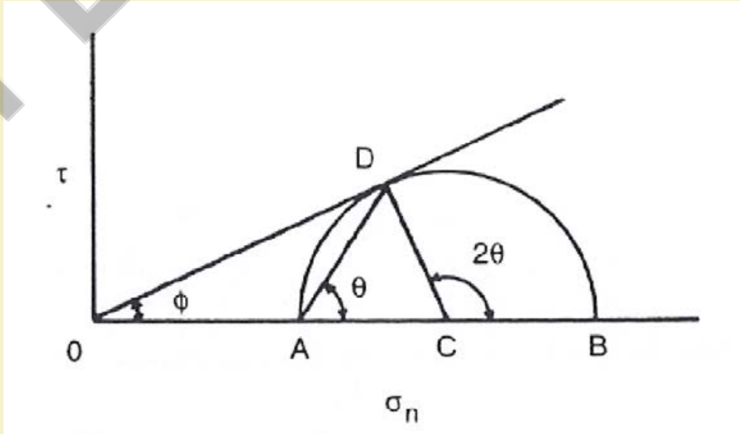
# SHEAR STRENGTH: summary

**Relationship between θ and ø :** From the figure angle DCO = 180 -2 θ

In triangle ODC: Angle DOC = ϕ ODC = 90 deg OCD = 180 -2 θ and these angles summed to 180 deg,

$$\phi + 90 + 180 - 2\theta = 180$$

**Hence, θ = 45 + ø/2**





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# SHEAR STRENGTH: summary

$$\tau_f = c + \sigma \tan\varphi$$

For sand c = 0 and for Clay ϕ = 0

$$\tau_f = c' + \sigma' \tan\varphi'$$

$$\sigma_1 - \sigma_3 = 2c\cos\emptyset + (\sigma_1 + \sigma_3)\sin\emptyset$$

$$\sigma_1' - \sigma_3' = 2c\cos\emptyset' + (\sigma_1' + \sigma_3')\sin\emptyset'$$




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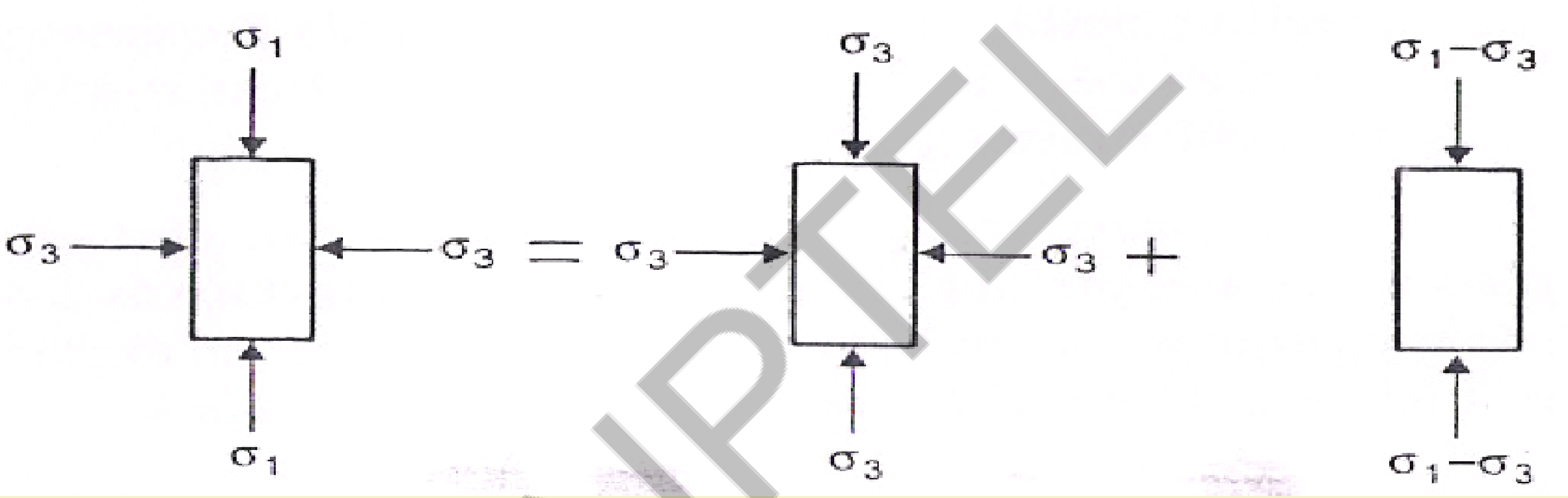
# Determination of Shear Strength Parameters

1. Direct Shear Test
2. Tri-axial Shear Test
3. Unconfined compression Test
4. Vane Shear Test

# SHEAR STRENGTH: summary





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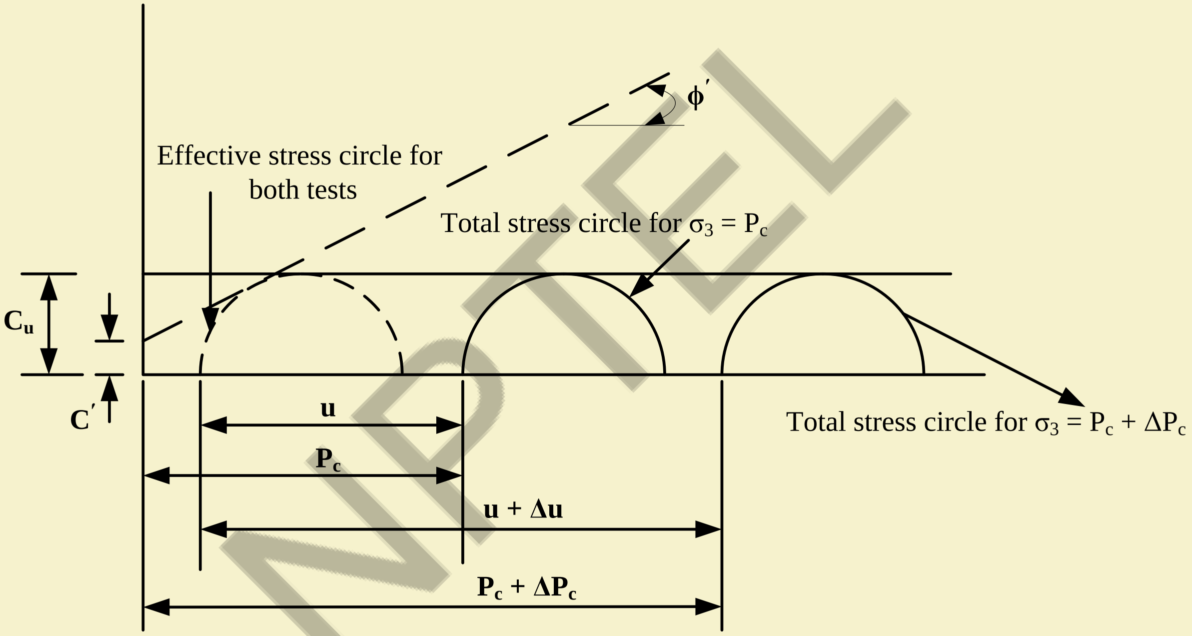
# Types of Tri-axial Test

1. Unconsolidated Undrained Test
2. Consolidated Undrained Test
3. Consolidated Drained Test

# SHEAR STRENGTH: summary

## Undrained shear , cu





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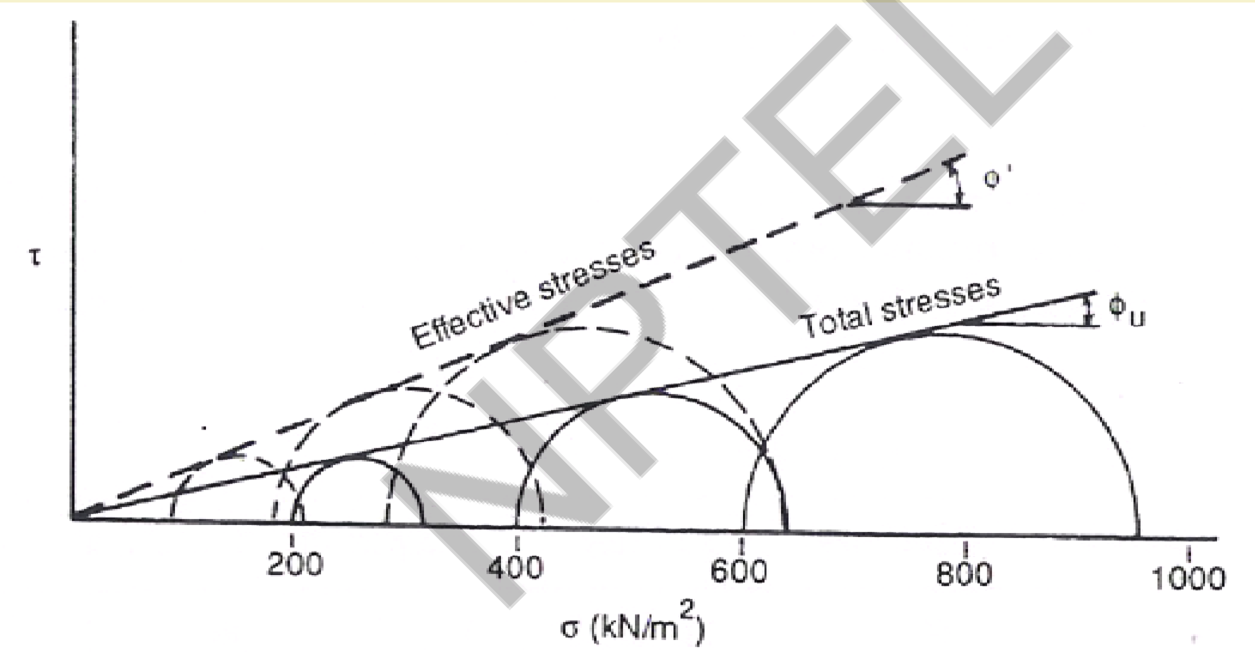
# SHEAR STRENGTH: summary

CU test results





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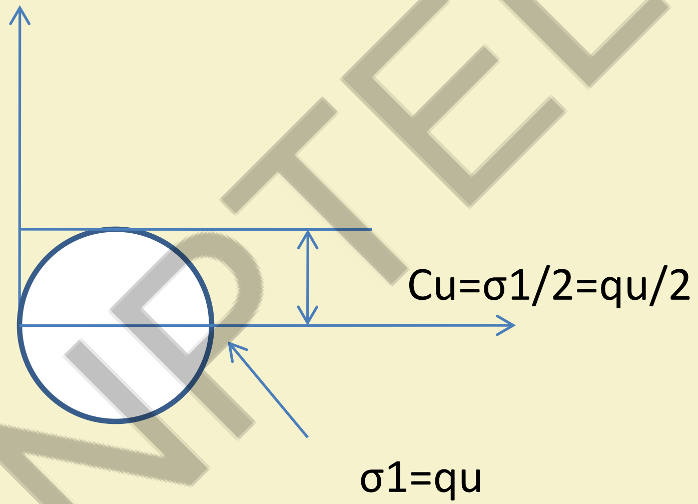
# SHEAR STRENGTH: summary

## Unconfined compression test:







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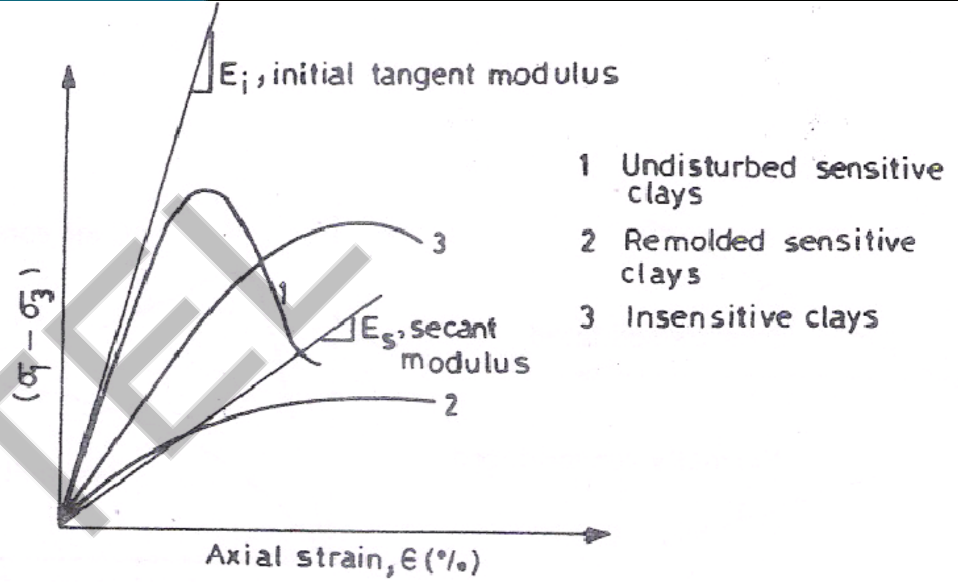
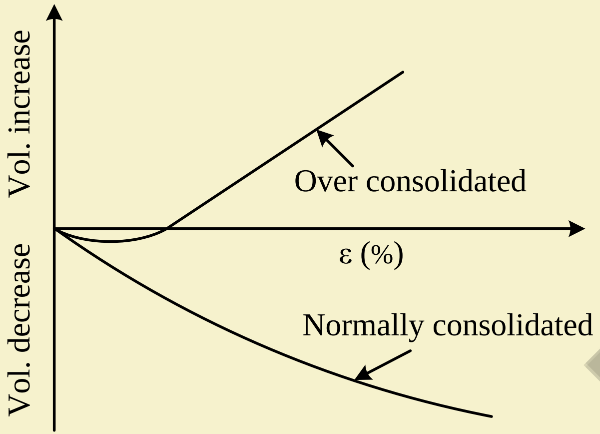
# SHEAR STRENGTH: summary

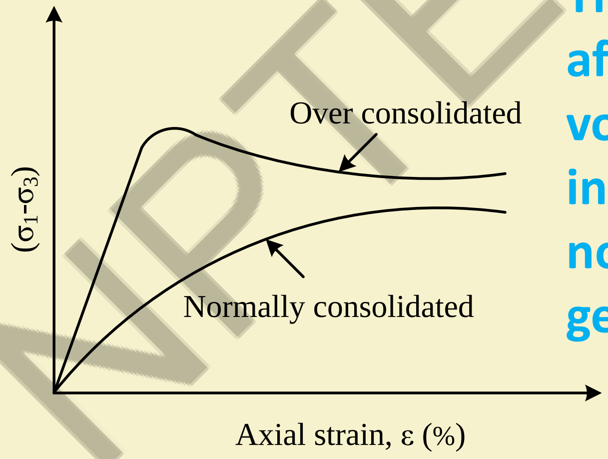
Stress-strain relationships can be obtained from the results of tri-axial tests or direct shear test In the direct shear tests, the results are plotted in the form of shear displacement (x-axis) versus shear stress (y-axis). In the tri-axial test the deviator stress is plotted as ordinate against the axial strain in percent. The unconfined compression test data are shown by plotting axial stress against axial strain




# SHEAR STRENGTH: summary

**The overconsolidated clay shows a greater strength than a normally consolidated clay and has a pronounced peak which occurs quite early. The stress falls off as the strain increases – a phenomenon called wok softening.**

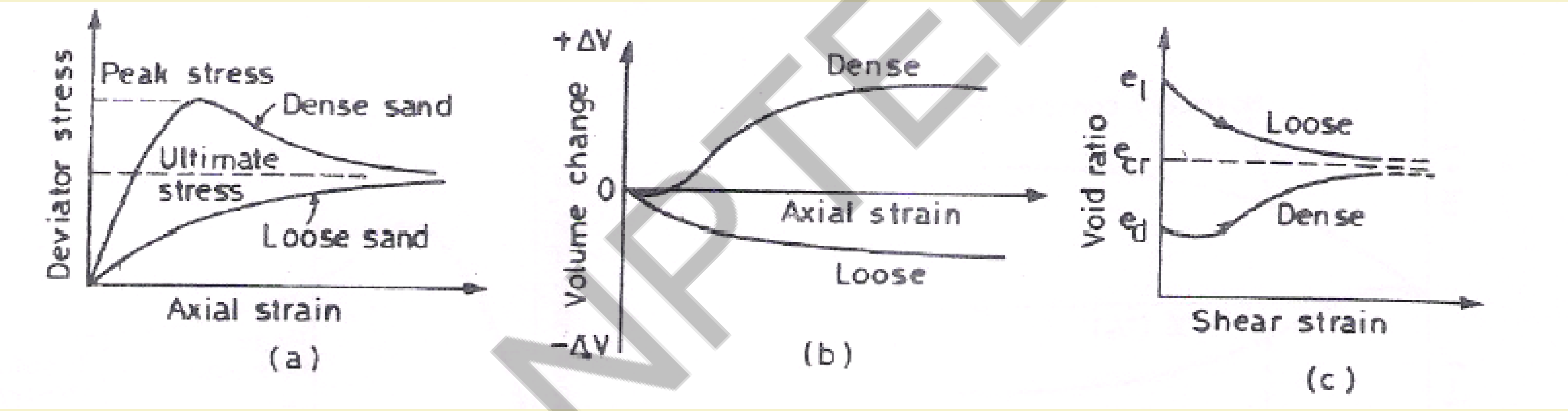



**The overconsolidated clay, after a small initial decrease in volume, shows a volume increase upon shear while the normally consolidated clays gets compressed when sheared**

# SHEAR STRENGTH: summary

Stress – strain and volume change behaviour of sand:



# SHEAR STRENGTH: summary

## Vane shear Test:

$$T = \pi c_u \left( \frac{d^2 h}{2} + \frac{d^3}{6} \right)$$

Shear booth at top and bottom

$$T = \pi c_u \left( \frac{d^2 h}{2} + \frac{d^3}{12} \right)$$

Shearing at bottom only

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# SOIL MECHANICS/GEOTECHNICAL ENGINEERING I

## COMPRESSIBILITY OF SOILS


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# Consolidation - Introduction

The effect occurs for saturated fine grained soils

A large wheel load rolling along a roadway resting on clay will cause an immediate settlement which is recoverable once the wheel is passed

If the same load is applied permanently there will in addition be consolidation





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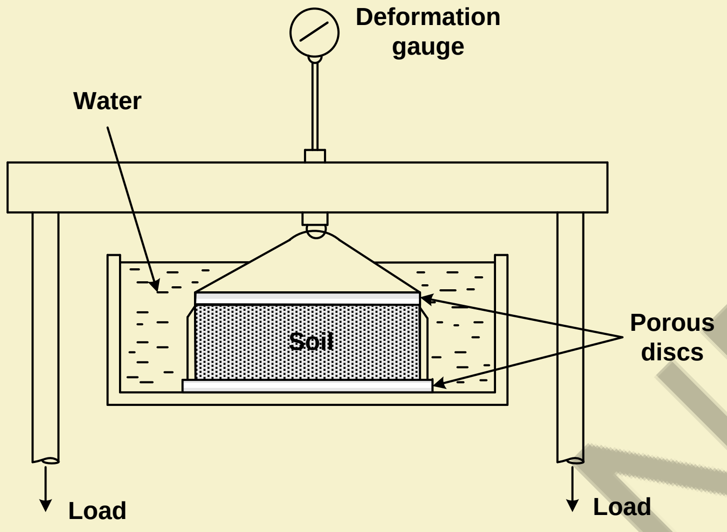
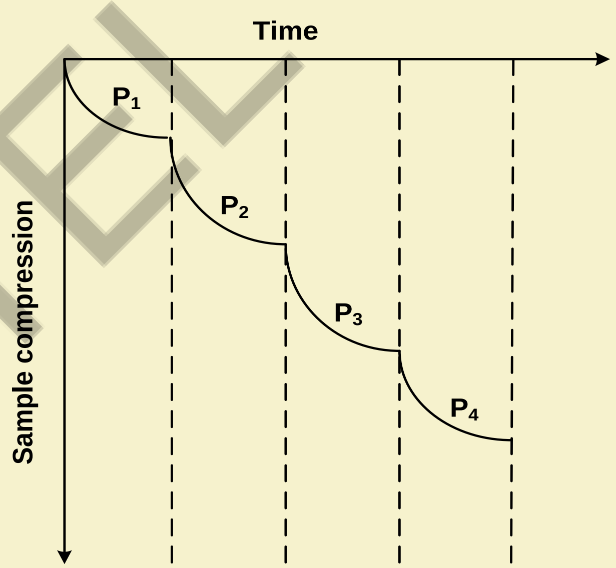
# Consolidation - Test




(a) Consolidation apparatus




(b) Typical test results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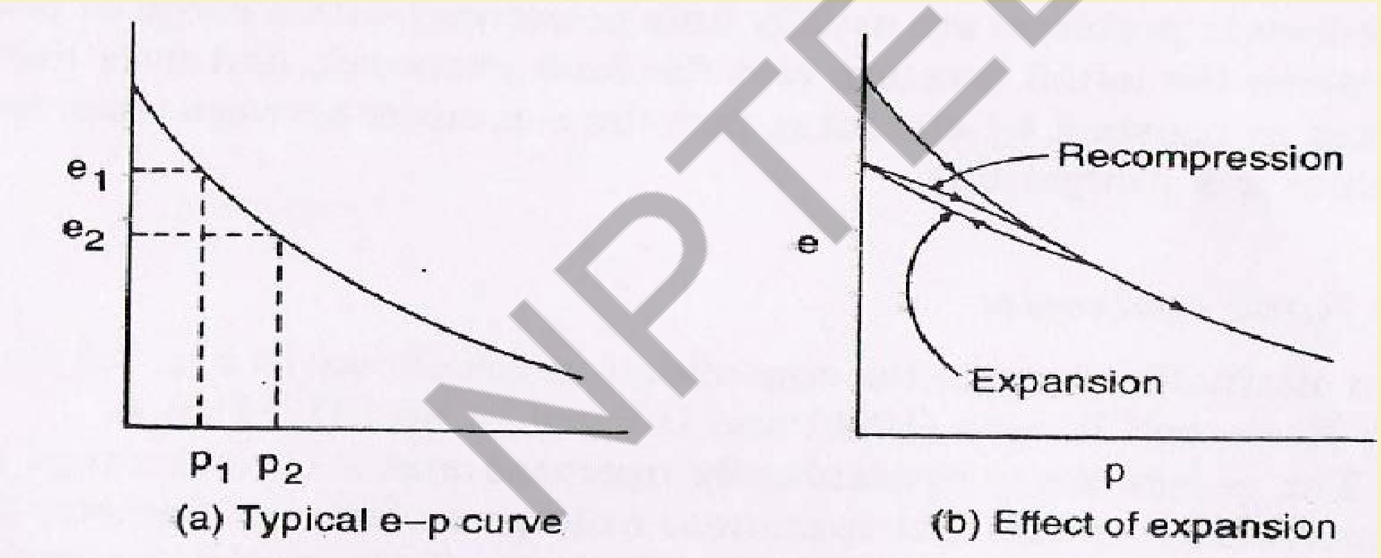
# COMPESSIBILITY OF SOILS

## e-p relationship







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# Coefficient of Volume compressibility

The volume change per unit of original volume constitutes the volumetric change. If a mass of soil of volume V1 is compressed to a volume V2, the assumption is made that the change in volume is caused by a reduction in the volume of voids.

$$\text{volumetric change} = \frac{V_1 - V_2}{V_1} = \frac{(1+e_1)-(1+e_2)}{(1+e_1)} = \frac{(e_1 - e_2)}{(1+e_1)}$$

Where e1 = void ratio at pressure p1 and e2 = void ratio at pressure p2





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# COMPESSIBILITY OF SOILS

mv = volumetric change/unit of pressure increase, If H1 is the original thickness and H2 is the final thickness,

$$volumentric\ change = \frac{V_1 - V_2}{V_1} = \frac{H_1 - H_2}{H_1}\ (AREA\ IS\ ASSUMED\ TO\ BE\ CONSTANT)$$

$$= \frac{e_1 - e_2}{1 + e_1}$$

Also $$a = \frac{e_1 - e_2}{dp}$$

$$volumetric\ change = \frac{a\ dp}{(1 + e_1)}$$

Hence, $$m_v = \frac{a\ dp}{(1 + e_1)} \frac{1}{dp} = \frac{a}{(1 + e_1)} \quad m^2/_{MN}$$

# COMPESSIBILITY OF SOILS

For most practical engineering problems mv values can be calculated for a pressure increment of 100 kN/m2 in excess of the present overburden pressure at the same depth

Once the coefficient of volume decrease has been obtained we know the compression/unit thickness/unit pressure increase. It is therefore an easy matter to predict the total consolidation settlement of a clay layer of thickness H:

$$total\ settlement = \rho_c = m_v\, dp\, H$$





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# Typical values of mv for different soils

| Soil | mv(m2/MN) |
|---|---|
| Peat | 10.0-2.0 |
| Plastic clay (normally consolidated alluvial clays) | 2.0-0.25 |
| Stiff clay | 0.25-0.125 |
| Hard Clay (boulder clays) | 0.125-0.0625 |





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# COMPESSIBILITY OF SOILS

The tabulated results were obtained from a consolidation test on a sample of saturated cay, each pressure increment having been maintained for 24 hrs.

After it had expanded for 24 hrs the samples was removed from the apparatus and found to have moisture content of 25%. The particle specific gravity of the soil was 2.65.

Plot void ratio to effective pressure curve and determine the value of the coefficient of volume change for a pressure range of 250-350 kN/m2

| Pressure (kN/m2) | Thickness of sample after consolidation (mm) |
|---|---|
| 0 | 20 |
| 50 | 19.65 |
| 100 | 19.52 |
| 200 | 19.35 |
| 400 | 19.15 |
| 800 | 18.95 |
| 0 | 19.25 |





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Solution: w = 0.25, G = 2.65

e = w G = 0.25×2.65 = 0.662 this is the void ratio corresponding to a sample thickness of 19.25 mm

$$\frac{dH}{H_1} = \frac{de}{1+e_1}$$

or

$$de = \frac{(1+e_1)}{H_1} dH = \frac{1.662}{19.25} dH = 0.0865 dH$$





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# COMPESSIBILITY OF SOILS

| Pressure | H | dH | de | e |
|---|---|---|---|---|
| 0 | 20.0 | +0.75 | +0.065 | 0.727 |
| 50 | 19.65 | +0.40 | +0.035 | 0.697 |
| 100 | 19.52 | +0.27 | +0.023 | 0.685 |
| 200 | 19.35 | +0.10 | +0.009 | 0.671 |
| 400 | 19.15 | -0.10 | -0.009 | 0.653 |
| 800 | 18.95 | -0.30 | -0.026 | 0.636 |
| 0 | 19.25 | 0 | 0 | 0.662 |

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## COMPRESSIBILITY OF SOILS

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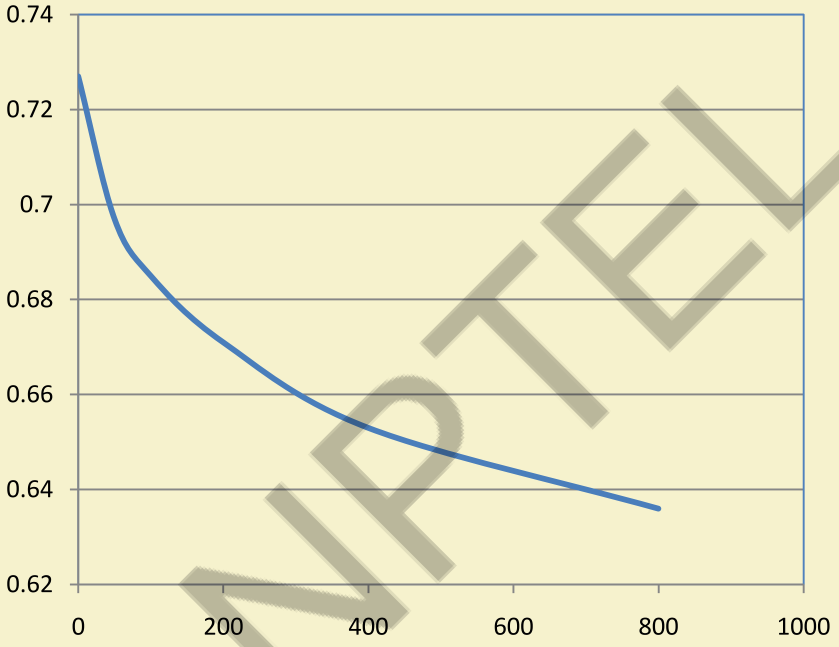
# COMPESSIBILITY OF SOILS



# COMPESSIBILITY OF SOILS

From the e-p curve:

e at 250 kN/m2 pressure = 0.666

e at 350 kN/m2 pressure = 0.658

$$a = \frac{de}{dp} = \frac{0.666 - 0.658}{100} = 0.00008\ kN/m2$$

$$m_v = \frac{a}{1 + e_1} = \frac{0.00008}{1.666} = 4.8 \times 10^{-5}\, m^2/_{kN}$$

# COMPESSIBILITY OF SOILS

The tabulated results were obtained from a consolidation test on a sample of saturated cay, each pressure increment having been maintained for 24 hrs.

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| 0 | 19.25 |





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# COMPESSIBILITY OF SOILS

Alternative method for determining mv:
mv can be expressed in terms of thickness

$$m_v = \frac{dH}{H_1}\frac{1}{dp} = \frac{1}{H_1}\frac{dH}{dp}$$

$\frac{dH}{dp}$ is the slope of the sample thickness vs pressure curve. Hence mv can be obtained by finding the slope of the curve at the required pressure and dividing by the original thickness. The thickness vs pressure curve is shown in Fig b. H at 250 kn/m2 = 19.28 mm and H at 350 kN/m2 = 19.19 mm.

$$m_v = \frac{19.28 - 19.19}{19.28 \times 100}\frac{m^2}{kN} = 4.7 \times 10^{-5}\frac{m^2}{kN}$$

If a layer of this clay, 20 m thick, subjected to this pressure increase then the consolidation settlement would have been: 0.000047x20x100x1000 mm = 96 mm




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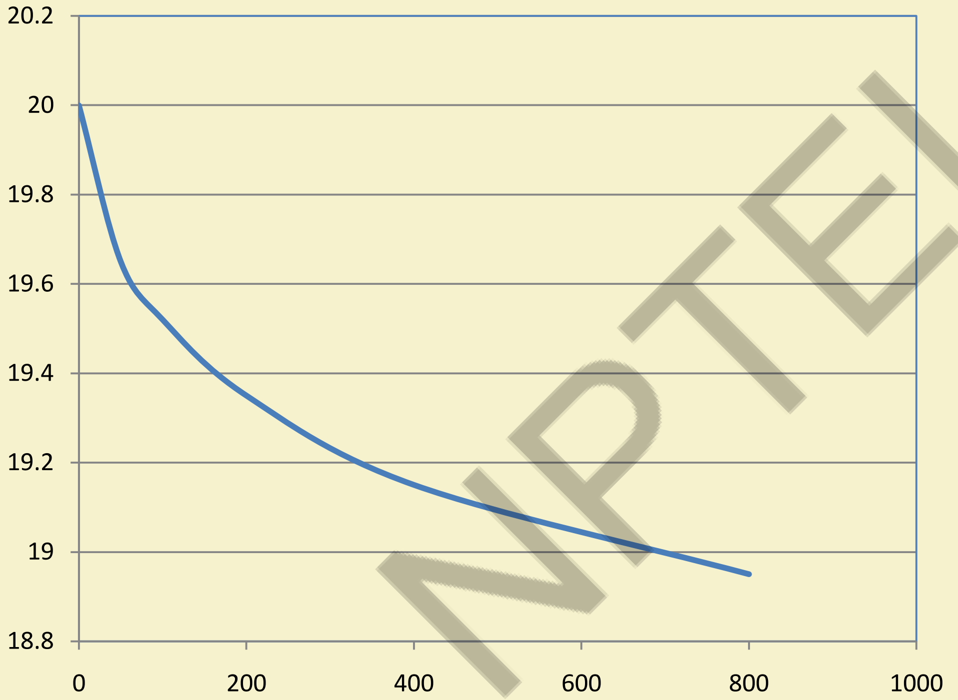
# COMPESSIBILITY OF SOILS



# COMPESSIBILITY OF SOILS

The practice of working back from the end of the consolidation test, i.e., from the expanded thickness, in order to obtain e-p curve is generally accepted as being the most satisfactory as there is little doubt that the sample is more likely to be fully saturated after expansion than at the start of the test.

It is, however, possible to obtain e-p curve by working from the original thickness.

Void ratio is given by:

$$e = \frac{V_v}{V_s} = \frac{V - V_s}{V_s} = \frac{A(H - H_s)}{AH_s} = \frac{H - H_s}{H_s}$$

Where A = c/s area of the sample , H = thickness of the sample and Hs = equivalent thickness of the solids

$$H_s = \frac{M_s}{\rho_w G_s A}$$





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# COMPESSIBILITY OF SOILS

Additional information for the problem: sample diameter 75 mm and thickness 20mm, mass of sample after removing from the consolidation apparatus at the end of the test and after drying = 135.6 gm

$$M_s = 135.6 \qquad A = \frac{\pi}{4} \times 75^2 = 4418\, mm^2$$

$$H_s = \frac{135.6 \times 1000}{2.65 \times 1 \times 4418} = 11.58\, mm$$

$$e = \frac{H - H_s}{H_s}$$

Using this equation void ratio at each pressure increment can be obtained.





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# COMPESSIBILITY OF SOILS

| Pressure (kN/m2) | Thickness(H, mm) | $e = \frac{H - H_s}{H_s}$ |
|---|---|---|
| 0 | 20.0 | 0.727 |
| 50 | 19.65 | 0.697 |
| 100 | 19.52 | 0.685 |
| 200 | 19.35 | 0.671 |
| 400 | 19.15 | 0.653 |
| 800 | 18.95 | 0.636 |
| | | |





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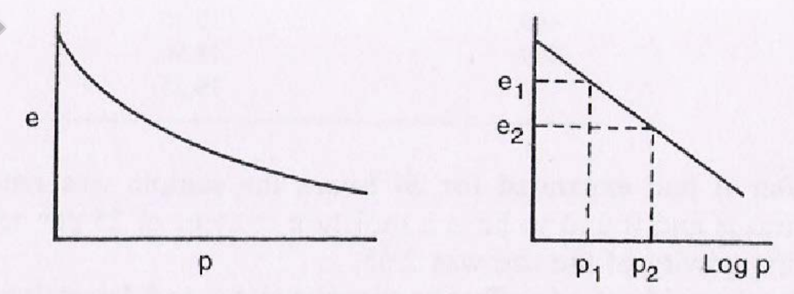
# COMPESSIBILITY OF SOILS

The clay is generally formed by the process of sedimentation from a liquid in which the soil particles were gradually deposited and compressed as more material was placed above them. The e-p curve corresponding to this natural process of consolidation is known as virgin consolidation curve. This curve is approximately logarithmic. If the values are plotted to a semi log scale the result is a straight line of equation:

$$e = e_0 - c_c \log_{10} \frac{p_0 + \Delta p}{p_0}$$



(a) Natural consolidation

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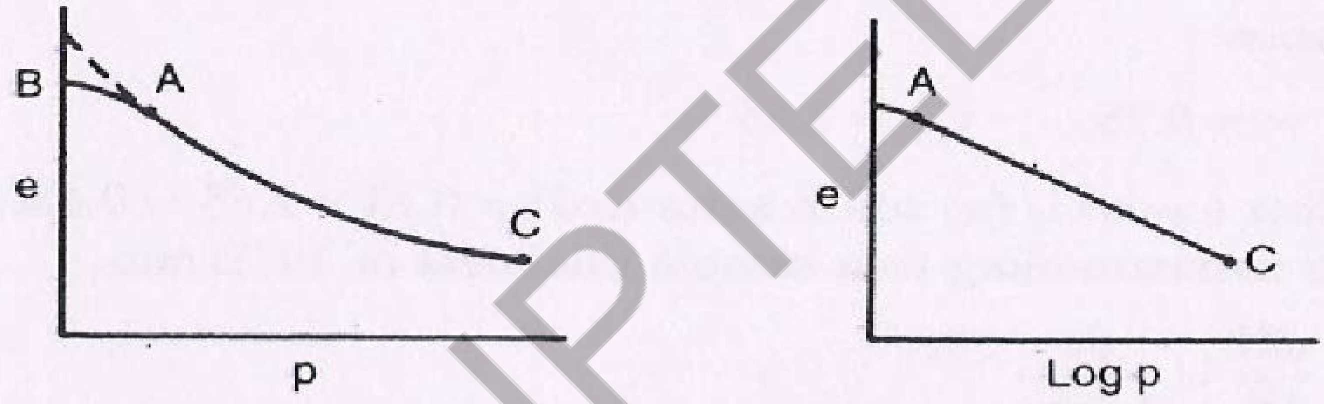
# COMPESSIBILITY OF SOILS




(b) Normally consolidated clay

## Compression curve for a Normally consolidated Clay:

A normally consolidated clay is one that has never experienced a consolidation pressure greater than that corresponding to its present overburden. The compression curve of such a soil is shown in the figure.

The clay was originally compressed, by the weight of the material above, along the virgin consolidation curve to some point A. Owing to the removal of pressure during sampling the soil has expanded to point B. Hence from B to A the soil is being recompressed whereas from A to C the virgin consolidation curve is followed. Semi-log plot corresponding to this is shown in the figure.

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## COMPRESSIBILITY OF SOILS

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## Compression curve for an over consolidated Clay:

An over consolidated clay is one which has been subjected to a pre consolidation pressure in excess of its existing overburden, the resulting compression being much less than for a normally consolidated clay. The semi-log plot is no longer a straight line and a compression index value for an over consolidated clay is no longer constant.





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# COMPRESSIBILITY OF SOILS

From the e-p curve it is possible to determine an approximate value for the pre-consolidation pressure with the use of a graphical method proposed by Casagrande (1936). First estimate the point of greatest curvature., A, then draw a horizontal line through A (AB) and tangent to the curve at A (AC). Bisect the angle BAC to give the line AD, and locate the straight part of the compression curve. Finally project the straight part of the curve upwards to cut AD at F. The point F then gives the value of the pre-consolidation pressure





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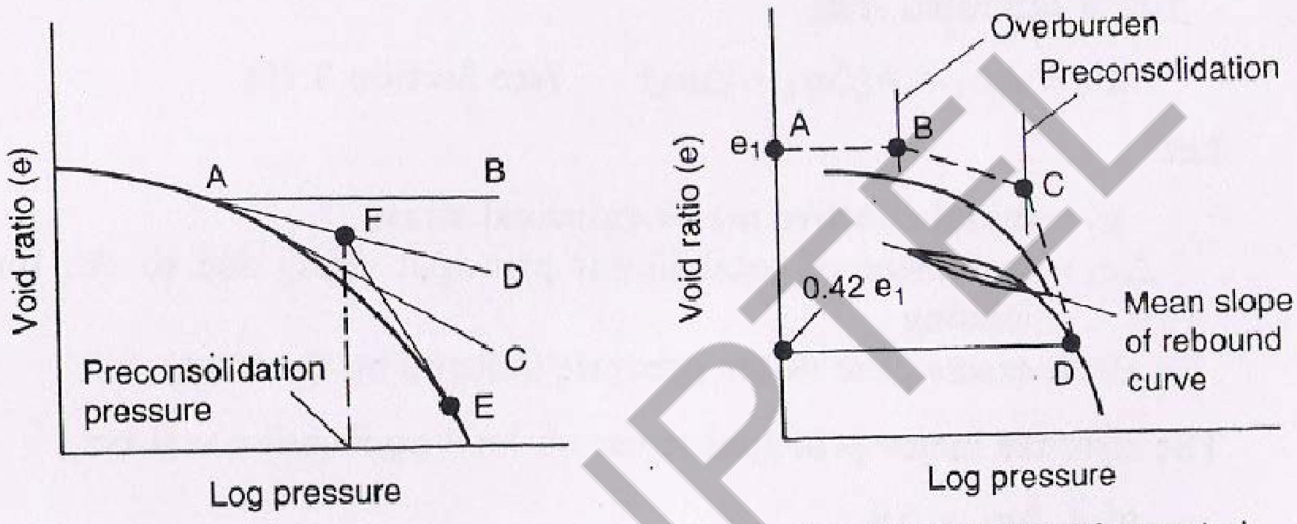
# COMPRESSIBILITY OF SOILS




(a) Graphical determination of preconsolidation pressure (Casagrande)

(b) Determination of corrected compression curve (Schmertmann)

# COMPRESSIBILITY OF SOILS

$$\frac{dH}{H_1} = \frac{e_1 - e_2}{1 + e_1}$$

$$dH = \frac{(e_1 - e_2)H_1}{1 + e_1} \qquad e_1 - e_2 = c_c \log_{10} \frac{p_2}{p_1}$$

$$\rho_c = \frac{c_c H_1}{1 + e_1} \log_{10} \frac{p_2}{p_1}$$

This equation is relevant only when the clay is being compressed 1st time and therefore can not be used for an over consolidated clay

**Approximate value of Compression index**: Terzaghi and peck (1948) have shown that there is an approximate relationship between the liquid limit of normally consolidated soil and its compression index. This relationship has been established experimentally and is:

$$c_c = 0.009(w_l - 10)$$

# COMPRESSIBILITY OF SOILS

A soft normally consolidated clay layer is 15 m thick with a natural moisture content of 45 percent. The clay has saturated unit weight 17.2 kN/m3, a particle specific gravity of 2.68, and a liquid limit of 65 percent. A foundation will subject the middle of the clay layer to a vertical stress increase of 10 kN/m2. Determine the approximate value of the consolidation settlement of the foundation if the ground water table is at the ground level.

# COMPRESSIBILITY OF SOILS

Solution: Initial vertical stress at the middle of the layer
= (17.2-9.81) 15/2 = 55.4 kN/m2

Final effective vertical Stress = 55.4 + 10 = 65.4 kN/m2

Initial void ratio = e1 = w G = 0.45x2.68 = 1.21
Cc = 0.009 (65-10) = 0.495

$$\rho_c = \frac{0.495 \times 15}{1 + 1.21} \log_{10} \frac{65.4}{55.4} m = 240\, mm$$

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# COMPRESSIBILITY OF SOILS

Over consolidated – when $\sigma_c'$ is larger than $\sigma_0'$, the clay is known to be overconsolidated

When $$\sigma_0' + \Delta\sigma \langle \sigma_{pc}'$$

$$\rho_c = \frac{c_r H}{1+e_0} \log_{10}\left(\frac{\sigma_0' + \Delta\sigma}{\sigma_0'}\right)$$





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# COMPRESSIBILITY OF SOILS

When $\quad \sigma_0' + \Delta\sigma \rangle \sigma_{pc}'$

$$\rho_c = \frac{c_r H}{1+e_0}\log_{10}\left(\frac{\sigma_0' + (\sigma_{pc}' - \sigma_0')}{\sigma_0'}\right) + \frac{c_c H}{1+e_0}\log_{10}\left(\frac{\sigma_{pc}' + (\sigma_0' + \Delta\sigma - \sigma_{pc}')}{\sigma_{pc}'}\right)$$

Or $\quad \rho_c = \frac{c_r H}{1+e_0}\log_{10}\left(\frac{\sigma_{pc}'}{\sigma_0'}\right) + \frac{c_c H}{1+e_0}\log_{10}\left(\frac{\sigma_0' + \Delta\sigma}{\sigma_{pc}'}\right)$





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# Thank You!!

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# SOIL MECHANICS/GEOTECHNICAL ENGINEERING I

## COMPRESSIBIITY OF SOILS

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# Settlement

| Soil Type | Percent of $\rho_f$ due to $\rho$ |
|---|---|
| Sand | 70 – 90% |
| Stiff Clay | 40 – 60% |
| Soft Clay | 10 – 25% |





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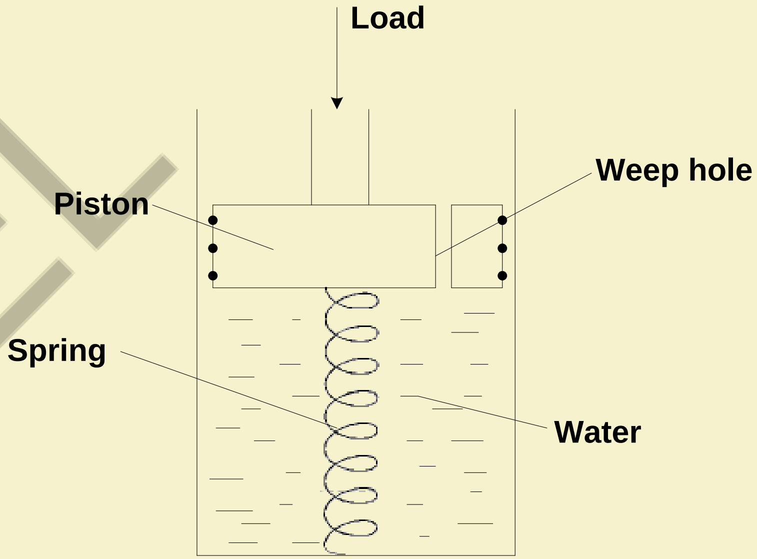
# Degree of Consolidation

The settlement of a foundation on cohesionless soil and the elastic settlement of a foundation in clay can be assumed to occur as soon as the load is applied. The consolidation settlement of a foundation on clay will only take place as water seeps from the soil at a rate depending upon the permeability of the clay




The model shown in the figure helps to understand consolidation process

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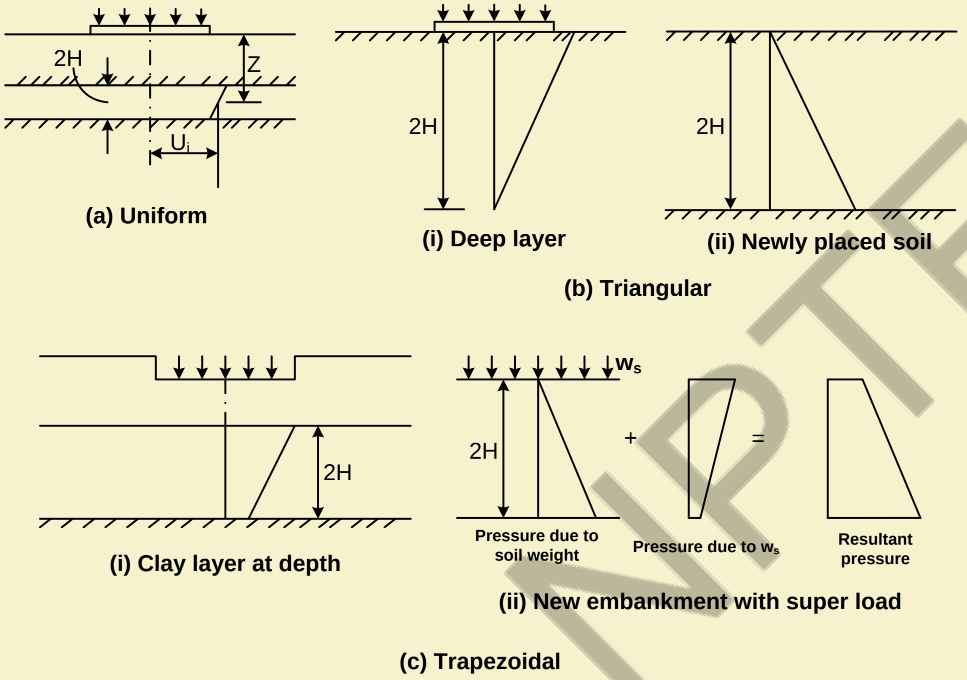
# COMPPRESSIBILITY OF SOILS




The degree of consolidation, U = (consolidation attained at time t)/ (total consolidation)





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# COMPPRESSIBILITY OF SOILS

Uniform distribution can occur in thin layers so that for all practical purposes ui is constant and equal to $\Delta\sigma$ at the centre of the layer.

The triangular distribution is found in a deep layer under a foundation where ui varies from a maximum value at the top to a negligible value at some depth below the foundation. The depth of this variation depends upon the dimension of the footing.





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# COMPPRESSIBILITY OF SOILS

A triangular distribution with ui = 0.0 at the top of the layer and ui = a maximum value at the bottom can occur for a newly placed layer of soil.

Trapezoidal distribution results from the quite common situation of a clay layer at some depth below the foundation. For a new embankment carrying superimposed load a reversed form of trapezoidal distribution is possible.





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# One dimensional Consolidation – Terzaghi's Theory

Assumptions:

- Soil is saturated
- The coefficient of permeability is constant
- Darcy's law of saturated flow applies
- The resulting compression is one dimensional
- Water flows in one direction
- Volume change are due solely to changes in void ratio, which are caused by corresponding changes in effective stress





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# COMPPRESSIBILITY OF SOILS

The expression for flow in saturated soil has been established before. The rate of volume change in a cube of volume dx.dy.dz

$$\left(k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} + k_z \frac{\partial^2 h}{\partial z^2}\right) dx\, dy\, dz$$

For one dimensional flow there is no component of hydraulic gradient in the x and y directions, and putting kz = k the expression become:

$$k \frac{\partial^2 h}{\partial^2 z} dx\, dy\, dz$$

The volume change during consolidation is assumed to be caused by changes in void ratio,

$$porosity, n = \frac{V_v}{V} = \frac{e}{1+e}$$


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# COMPPRESSIBILITY OF SOILS

Hence, $$V_v = dxdydz\frac{e}{1+e}$$

Another expression for rate of change of volume is therefore,

$$\frac{\partial}{\partial t}\left(dxdydz\,\frac{e}{1+e}\right)$$

Equating these two expressions: $$k\frac{\partial^2 h}{\partial z^2} = \frac{1}{1+e}\frac{\partial e}{\partial t}$$

The head, h, causing flow is the excess hydrostatic head caused by the excess pore water pressure, u

$$h = \frac{u}{\gamma_w}$$ and $$\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2} = \frac{1}{1+e}\frac{\partial e}{\partial t}$$

# COMPPRESSIBILITY OF SOILS

With one dimensional consolidation there are no lateral strain effects and the increment of applied pressure is therefore numerically equal (but of opposite sign) to the increment of induced pore pressure. Hence an increment of applied pressure dp will cause an excess pore water pressure of du (=-dp). Now

$$a = -\frac{de}{dp} = \frac{de}{du}$$





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# COMPPRESSIBILITY OF SOILS

Substituting for de

$$\frac{k}{\gamma_w}(1+e)\frac{\partial^2 u}{\partial z^2} = a\frac{\partial u}{\partial t}$$

$$c_v\frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t}$$

Where cv is the coefficient of consolidation and equals

$$\frac{k}{\gamma_w\, a}(1+e) = \frac{k}{\gamma_w\, m_v}$$




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# COMPPRESSIBILITY OF SOILS

In the theory Z is measured from the top of the consolidating layer and drainage is assumed at both the upper and lower surfaces. Thickness of the layer is 2H. The initial excess pressure , ui = -dp. The boundary conditions:

When Z = 0 u = 0,
When Z = 2H u = 0
When t = 0, u = ui

A solution for $c_v \dfrac{\partial^2 u}{\partial z^2} = \dfrac{\partial u}{\partial t}$ satisfying above $u_z = \sum_{m=0}^{m-\infty} \dfrac{2u_i}{M}\left(sin\dfrac{Mz}{H}\right)e^{-M^2T}$


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# COMPPRESSIBILITY OF SOILS

Where ui is the initial excess pore pressure, uniform over the whole depth

$$M = \frac{1}{2}\pi(2m+1)$$ where m is the positive integer varying from 0 to ∞

$$T = \frac{c_v t}{H^2}$$

Because of the drainage provision at the top and bottom of the layer , ui will immediately fall to zero at these points. With the mathematical solution it is possible to determine u at time t for any point within the layer. If these values of pore pressures are plotted a curve (isochrone)can be drawn though the points. The maximum excess pressure is seen to be at the center of the layer and for any point the applied pressure increment $\Delta\sigma_1 = u + \Delta\sigma_1'$

# COMPPRESSIBILITY OF SOILS

After a considerable time u will become equal to zero and Δσ1 will equal to Δσ1’. Plot of isochrone for different time interval is shown. For a particular point the degree of consolidation, Uz will be equal to

$$\frac{u_i - u_z}{u_i}$$

The mathematical expression of Uz is $U_z = 1 - \sum_{m=0}^{m-\infty} \frac{2}{m} sin\left(\frac{Mz}{H}\right) e^{-m^2 T}$

Average degree of consolidation U $U = \frac{2Hu_i - Area\ of\ isochrone}{2Hu_i}$





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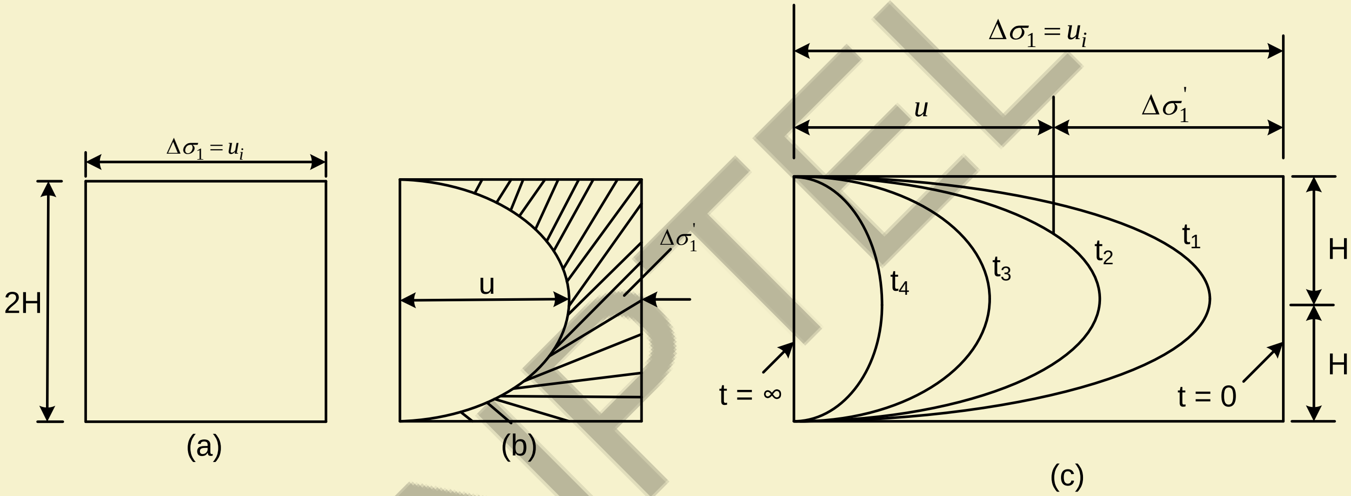
# COMPPRESSIBILITY OF SOILS



# COMPPRESSIBILITY OF SOILS

Mathematical expression for U is: $$U = 1 - \sum_{m=0}^{m-\infty} \frac{2}{M^2} e^{-M^2 T}$$





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