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COURSE NAME

TOPIC NAME

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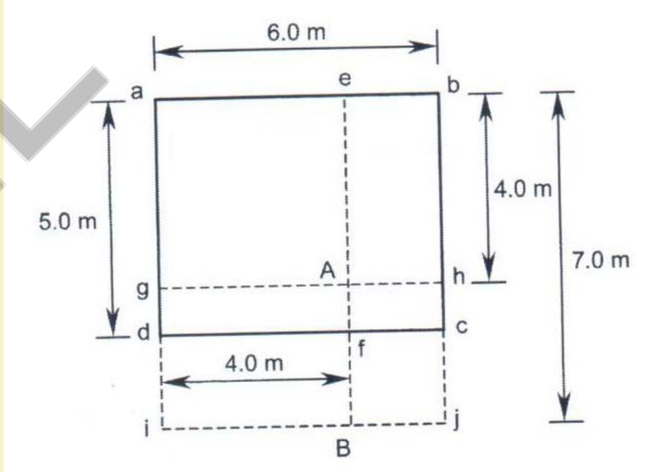
VERTICAL STRESS: APPLICATION

The uniform contact pressure under a rectangular footing of 6 m by 5 m is 200 kPa. Compute the vertical stress component under point A and B at a depth of 2 m (Aysen).



VERTICAL STRESS: APPLICATION

A square footing of size 4 m by 4 m is loaded with a contact pressure of 250 kPa. Determine the contact pressure at a depth of 2m, 4m, 8m and 12 m vertically below the center of the footing by Boussinesq point load formula, corner formula and approximate method.

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SOIL MECHANICS/GEOTECHNICL ENGINEERING I

SHEAR STRENGTH

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SHEAR STRNGTH

In many soil mechanics problems, the shear strength of the soil emerges as one of the most important characteristics. This problem group may include:

- Stability of slopes, i.e., cuts, embankments, earth dams, hill slopes etc
- Ultimate bearing capacity of a soil
- Lateral pressure against retaining walls, sheeting or bracing
- Friction develops by piles

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SHEAR STRNGTH

The shear strength of the soil may be attributed to three basic components:

- Frictional resistance to sliding between solid particles
- Cohesion and adhesion between the soil particles
- Interlocking and bridging of solid particles to resist deformation

It is neither easy nor practical to clearly delineate the effects of these components on the shear strength of the soil

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The components shown before may be influenced by:

- changes in the moisture content
- pore pressures
- structural disturbance
- fluctuation in the ground water table
- underground water movement
- stress history, time
- perhaps chemical action or environmental conditions

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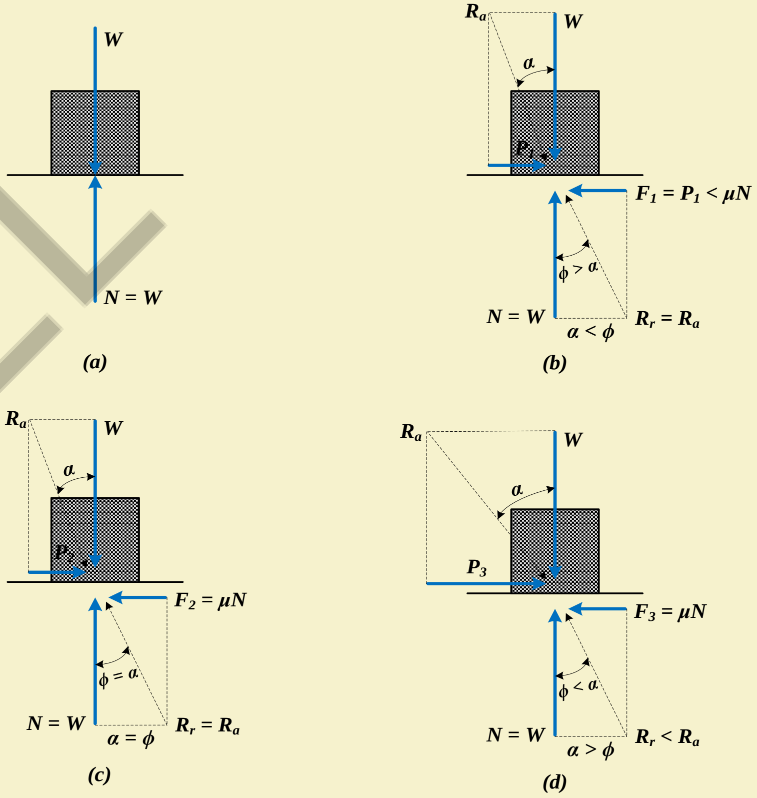
There are a number of tests both laboratory and field, that are used to obtain a measure of the shear strength of a given soil. A combination of the theoretical with the experimental efforts provides the background and tools for the basic understanding of phenomenon related to the shear strength of the soils

SHEAR STRNGTH

With some appropriate modifications, many of the concepts used in some of the basic mechanics courses can be employed to explain various phenomenon in soils

No horizontal force, W = N
Along with W small P, block will not move but Ra makes an angle, α with P1, called angle of obliquity of force P1
Increase horizontal force to P2, frictional force reaches F2, and it reaches to a maximum value when α is equal to the angle of friction, Ø



SHEAR STRNGTH

When $\alpha = \emptyset$ condition of impending motion exists.

$0 < \alpha < \emptyset$ No slip or sliding occurs

At the point of impending motion $F = \mu N$

Coefficient of friction μ is independent of the area of contact strongly dependent on the nature of the surface in contact, type of material, the condition of the surface and so on.

If the horizontal force is increased to a value P3 as shown such than α is greater than Ø, the block will start sliding. The frictional force cannot exceed the value given by $F = \mu N$ and therefore the block accelerate to the right

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From the condition of impending motion one can obtain, $tan\emptyset = \frac{F}{N} = \mu$

The frictional resistance in sands and other chesionless granular materials resembles the above. However, there are some important differences:

Frictional resistance in soils consists of both sliding and rolling friction, coupled with a certain degree of interlocking of the solid particles.

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In addition, various other factors, which influence the shear strength are:

- Degree of saturation,
- particle size and shape,
- consistency, or inter granular pressure

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In a plane passing through an element of mass that has been subjected to external loading both normal and shear stress act.

Stresses normal to a plane are designated by σ and those parallel to any given by τ.
There will be total nine components

Thee planes at right angles to each other where only normal stresses act, those planes are called as principal planes and stresses are known as principal stresses

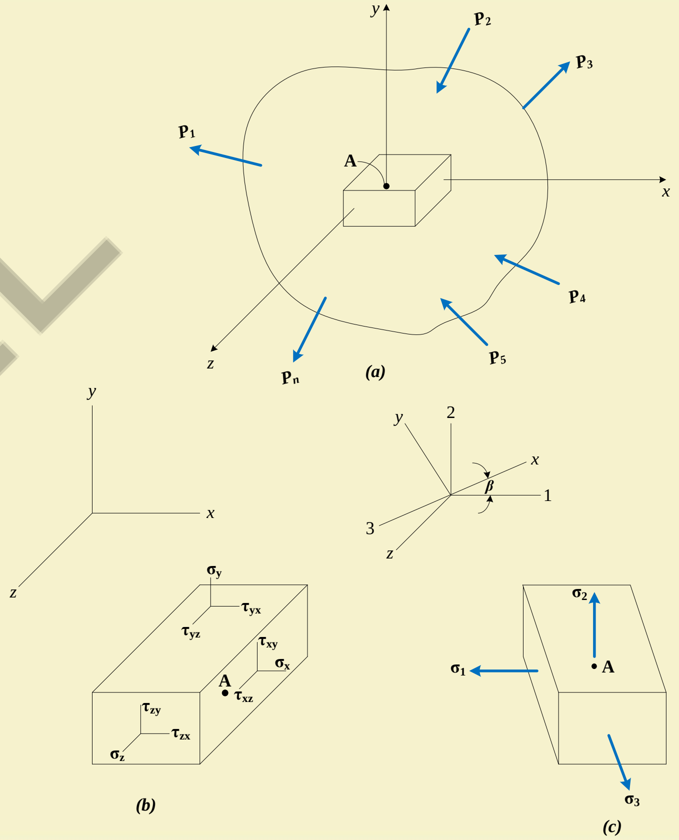

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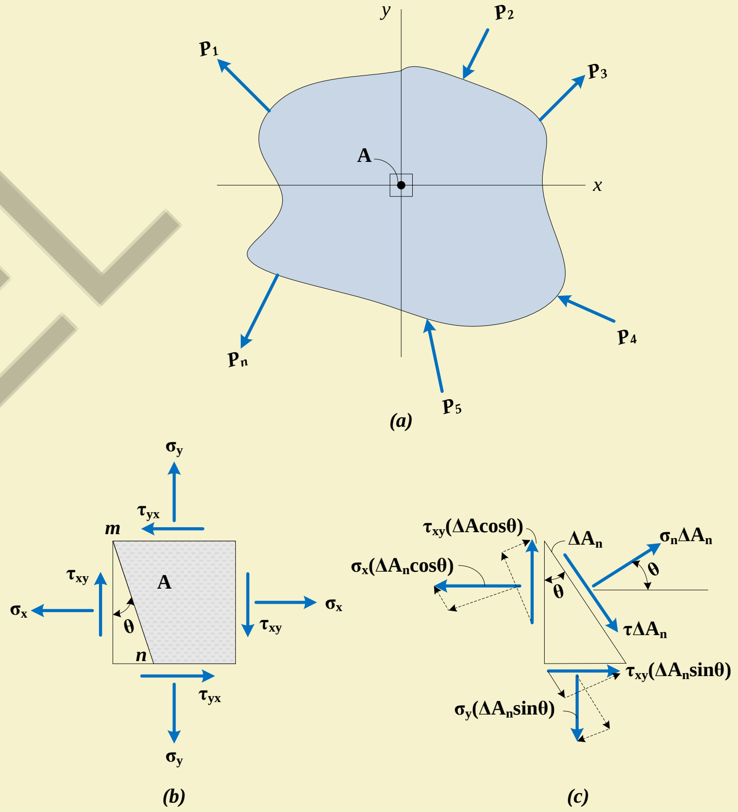
Illustration using biaxial stresses. mn plane oriented θ with respect to y axis area of inclined plane An, area of vertical and horizontal faces of the element, An cos θ and An sinθ. Now summing forces in the direction normal to the plane:

$$\sigma_n A_n = \sigma_x A_n cos\theta cos\theta - \tau_{xy} A_n cos\theta sin\theta + \sigma_y A_n sin\theta sin\theta - \tau_{xy} A_n sin\theta cos\theta$$

Which reduces to

$$\sigma_n = \sigma_x cos^2\theta + \sigma_y sin^2\theta - 2\tau_{xy} sin\theta cos\theta$$



SHEAR STRNGTH

After arranging the expression in terms of double angle the expression for normal stress at the plane can be expressed as given by,

$$\sigma_n = \frac{(\sigma_x + \sigma_y)}{2} + \frac{(\sigma_x - \sigma_y)}{2} cos2\theta - 2\tau_{xy} sin2\theta$$

Summing forces in the direction parallel to the plane:

$$\tau_n A_n = \sigma_n A_n cos\theta sin\theta + \tau_{xy} A_n cos\theta cos\theta - \sigma_y A_n sin\theta cos\theta - \tau_{xy} A_n sin\theta sin\theta$$

Simplifying and expressing in double angles it reduces to:

$$\tau_n = \frac{(\sigma_x - \sigma_y)}{2} sin2\theta + \tau_{xy} cos2\theta$$

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The orientation of principal plane is determined by setting $\frac{d\sigma_n}{d\theta} = 0$

i.e., $-(\sigma_x - \sigma_y)sin2\theta - 2\tau_{xy}cos2\theta = 0$

Or

$$tan2\theta = \frac{-\tau_{xy}}{(\sigma_x - \sigma_2)/2}$$

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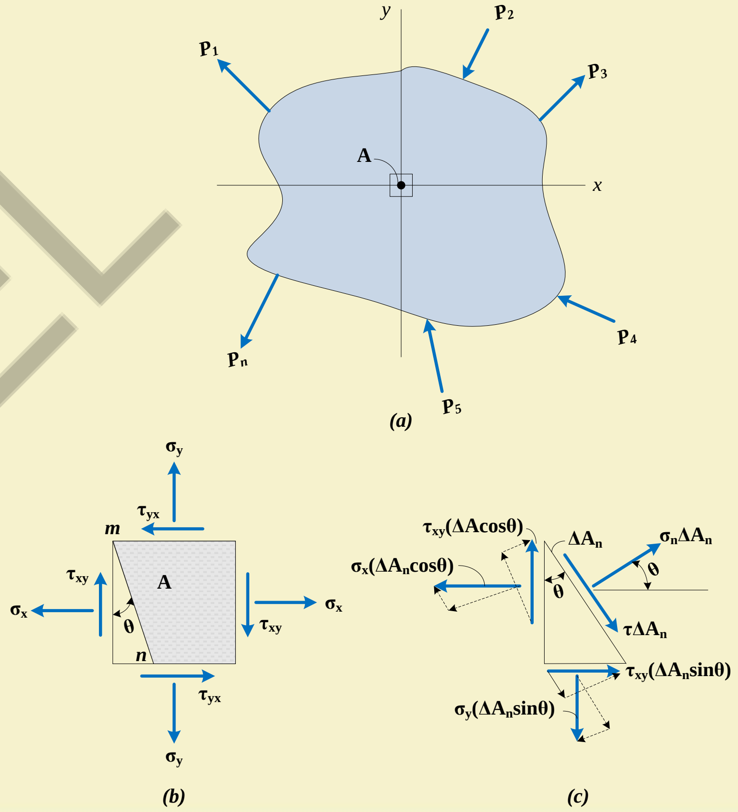
SHEAR STRENGTH

$$\sigma_n = \frac{(\sigma_x + \sigma_y)}{2} + \frac{(\sigma_x - \sigma_y)}{2} cos2\theta - 2\tau_{xy} sin2\theta$$

$$\tau_n = \frac{(\sigma_x - \sigma_y)}{2} sin2\theta + \tau_{xy} cos2\theta$$

$$tan2\theta = \frac{-\tau_{xy}}{(\sigma_x - \sigma_2)/2}$$



SHEAR STRENGTH

Mohr circle of Stress: Te expression for the normal and shear stresses cam be represented graphically by a useful device known as Mohr's circle for stress. It can be shown that locus of state of stress at any point is a circle. Squaring expression of σ_n and τ_n and rearranging the terms one can get

$$\left(\sigma_n - \frac{\sigma_x + \sigma_y}{2}\right) + {\tau_n}^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2$$

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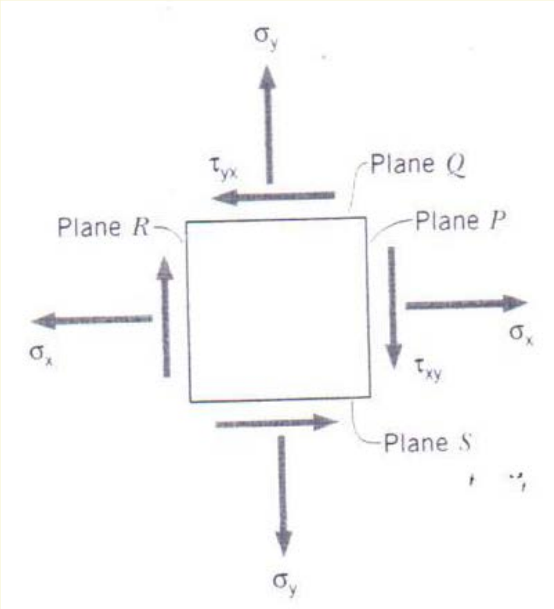
SHEAR STRENGTH

The expression is an equation of circle with center at $(\sigma_x + \sigma_y)/2,\ 0$ and radius equal to

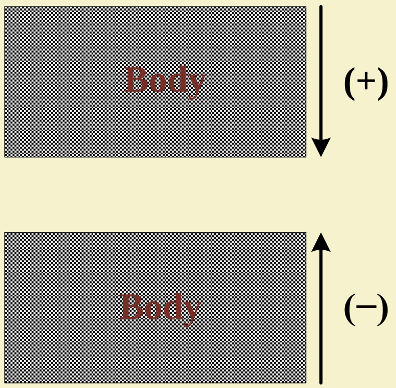
$$\sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$



Steps in construction of Mohr Circles:

The normal stresses are plotted as horizontal coordinates. The tensile stresses are considered positive (plotted right to the origin); the compressive stresses are considered negative (plotted to the left of he origin). In soil mechanics compressive stresses are taken positive

SHEAR STRNGTH



The shear stresses are plotted as vertical coordinates. Positive shear stresses are plotted above the origin and negative shear stresses are plotted below the origin.

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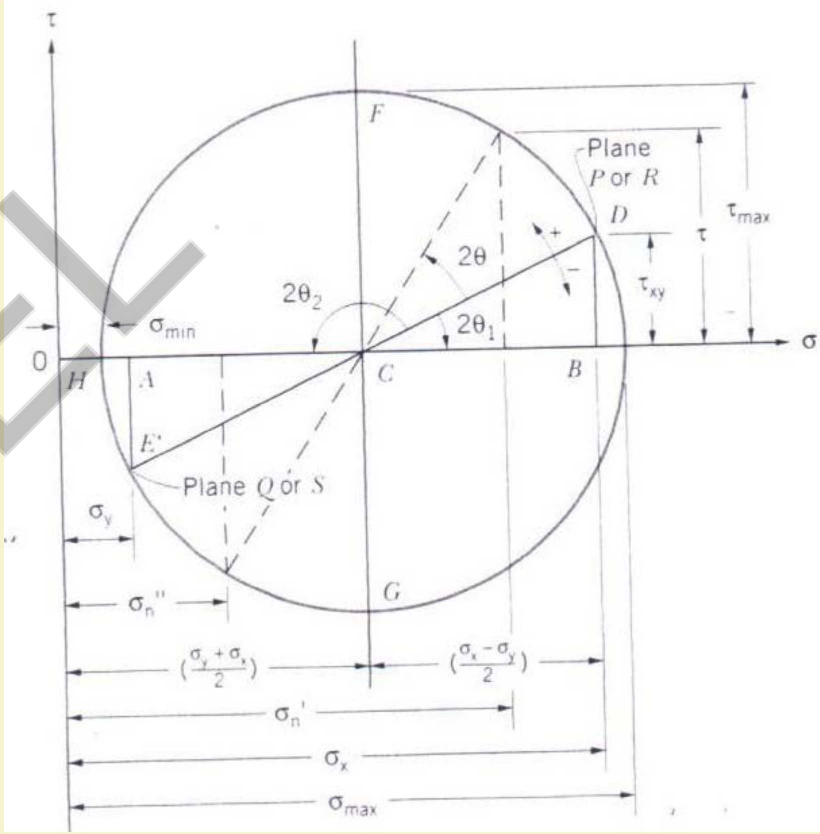
SHEAR STRENGTH

The positive angles on the circle are obtained when measured in the counterclockwise sense; negative on the circle are obtained in the clockwise sense. An angle of 2θ on the circle corresponds to an angle θ on the element



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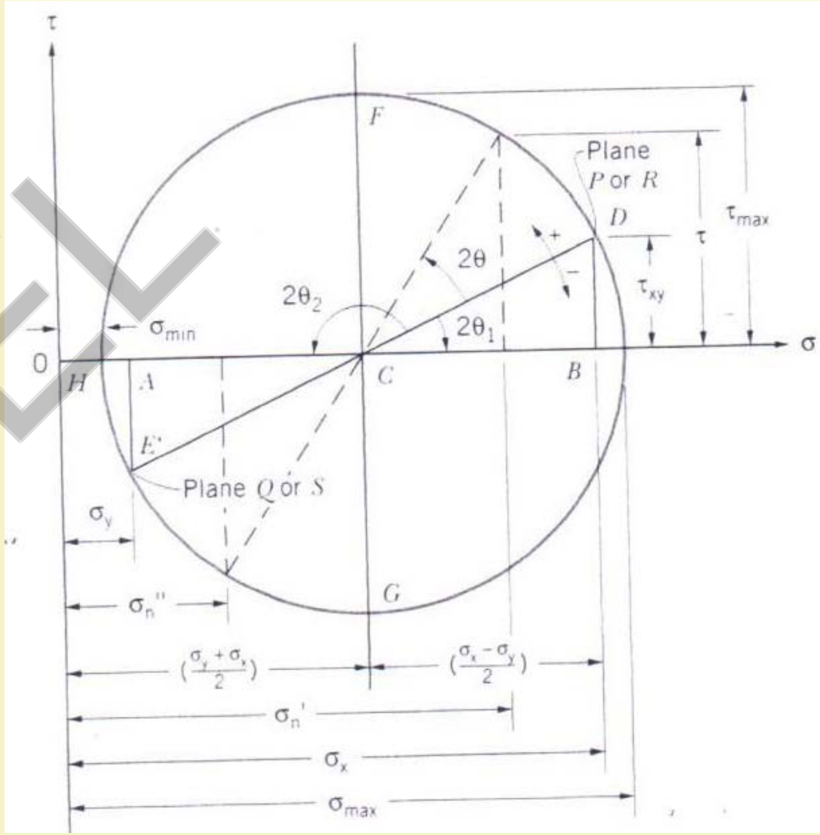
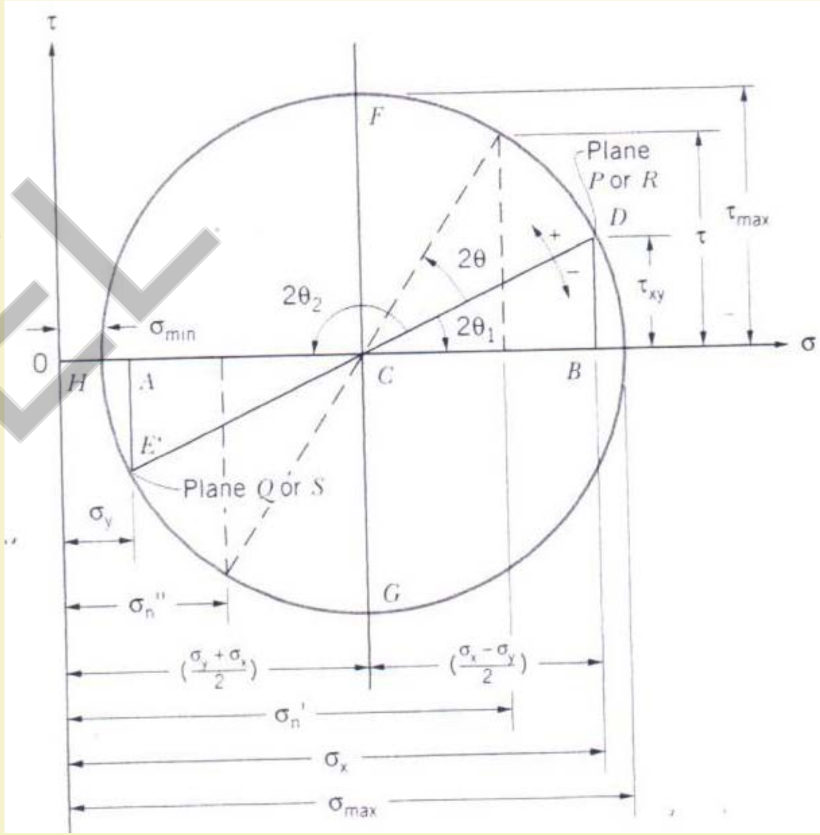
SHEAR STRENGTH

Figure shows the Mohr's circle for the assumed stresses at the stressed point assuming σx>σy>0. The normal and shear stresses on vertical plane P give the coordinates of one point on the circle, as indicated as D. Plane R also vertical and 180 deg from P, is located at 2θ =360 deg on the circle from P resulting in the same point on the circle. Similarly the stresses on the horizontal planes Q and S plot a different single point E



SHEAR STRENGTH

This shows that the stresses on parallel planes of an element oriented at any angle are correspondingly equal. The point of intersection of the line DE connecting these two points and the horizontal axis σn gives the location of the center of the circle. The diameter of the circle is DE.



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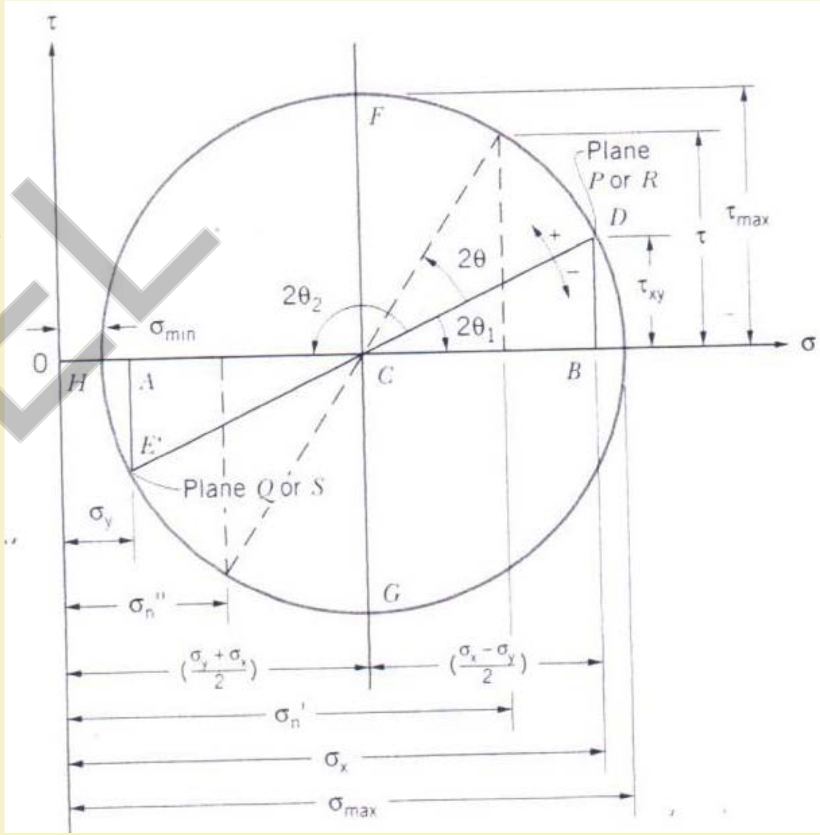
SHEAR STRENGTH

The maximum shear stress is apparent to be equal to the radius of the circle. The orientation of this stress with respect to the original plane can either be measured or calculated with the help of trigonometry. The principal stresses σmax and σmin can likewise be spotted and determined easily, and orientation can easily be measured or calculated. Furthermore, on the principal planes the shear stresses are zero



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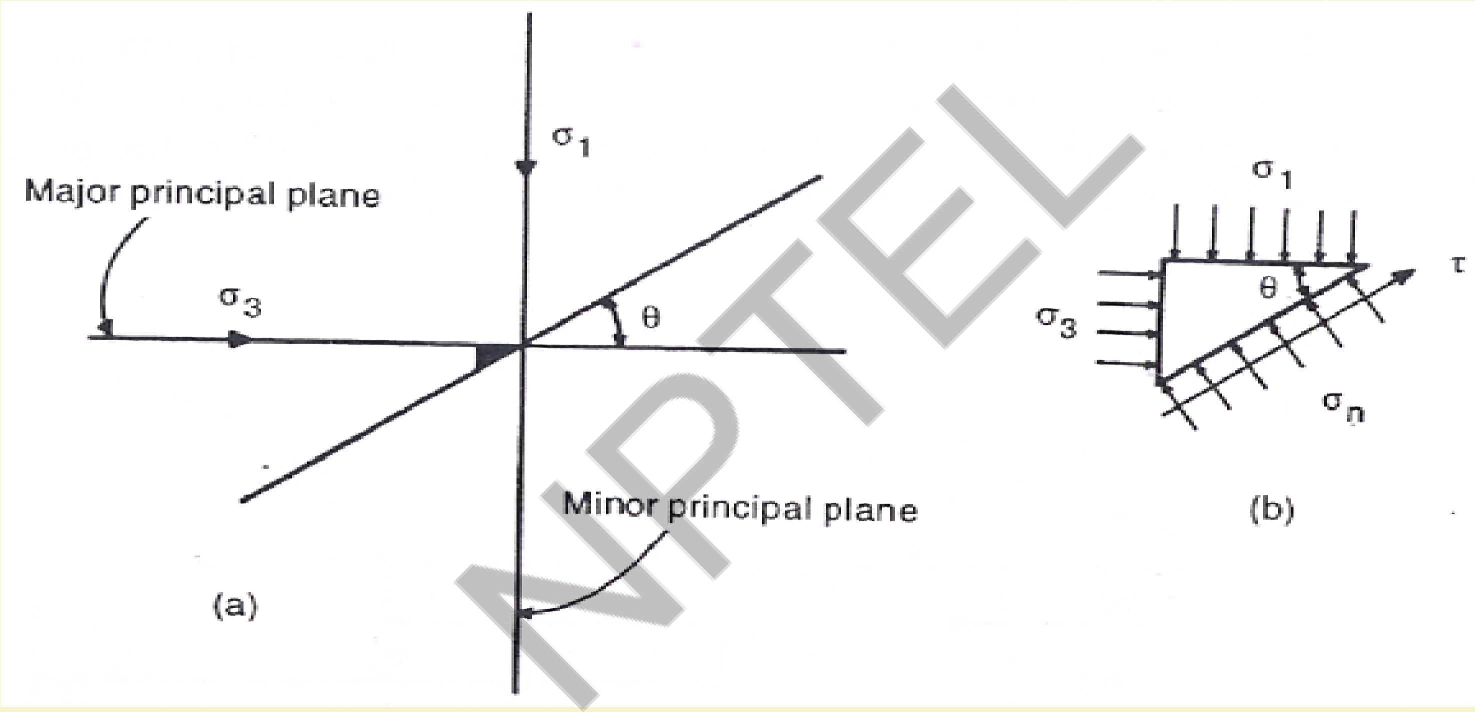
SHEAR STRENGTH



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SHEAR STRENGTH

A number of additional basic relationships which can be obtained from Mohr's circle:

- The maximum shear stress is equal to the radius of the circle. Furthermore the maximum shear stresses are on planes that makes an angle 45 deg with the principal plane

- The resultant stress on any plane has a magnitude of $\sqrt{\tau^2 + \sigma^2}$ and its angle of obliquity is equal to $tan^{-1}(\tau/\sigma)$

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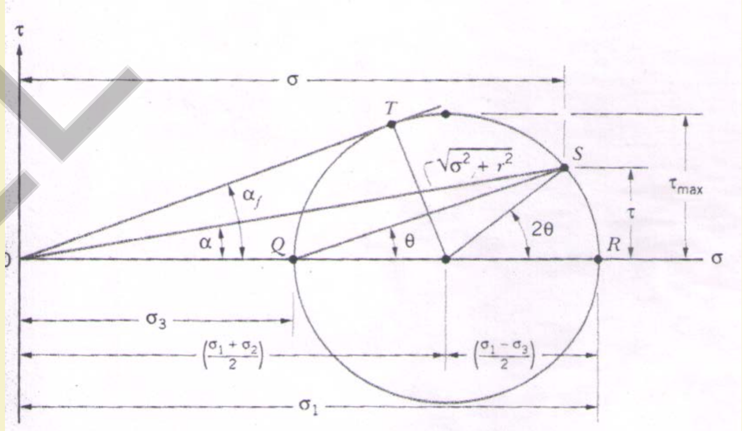
SHEAR STRENGTH

- The maximum angle of obliquity is constructed by the line OT such that the line is tangent to the circle at point T. Correspondingly, the stresses, normal and shear, that correspond to point T on the circle represent the stresses on the plane of maximum obliquity. It may be noted that shear stress on this plane is less than τ_{max} . Thus it may further be noted that the slippage occurs at the point of maximum obliquity and not at the angle where τ_{max}



SHEAR STRENGTH

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SOIL MECHANICS/GEOTECHNICAL ENGINEERING I

SHEAR STRNGTH: application

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SHEAR STRNGTH: application

State of plane stress at a point in a body is shown in the figure. Determine the principal stresses and the orientation of the planes on which they act

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SHEAR STRENGTH: application

Given a major principal stress of 400 kPa (compressive) and a minor principal stress of 100 kPa (compressive), determine the maximum in plane shear stress and the orientation of the plane on which they act.

SHEAR STRENGTH: application

Given the state of plane stress shown in Figure, compute

(i) The stress acting on an element that is rotated 20 deg counterclockwise from the x-y axes as shown

(ii) The major and minor principal stresses and the orientation of the principal stress direction

(iii) The stress acting on an element whose faces are aligned with the panes of maximum shear stress

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SEAR SENGTH

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Principal Plane: A plane that is acted upon by a normal stress only is known as a principal plane, there is no tangential or shear stress present.

Principal Stress: The normal stress acting on principal plane is referred to as a principal stress. At every point in a soil mass, applied stress system that exists can be resolved into three principal stresses that are mutually orthogonal. The principal planes corresponding to these principal stresses are called major, intermediate and minor principal planes and are so named from the consideration of the principal stresses that act upon them. The principal stress σ is known as the major principal stress acts on the major principal plane.

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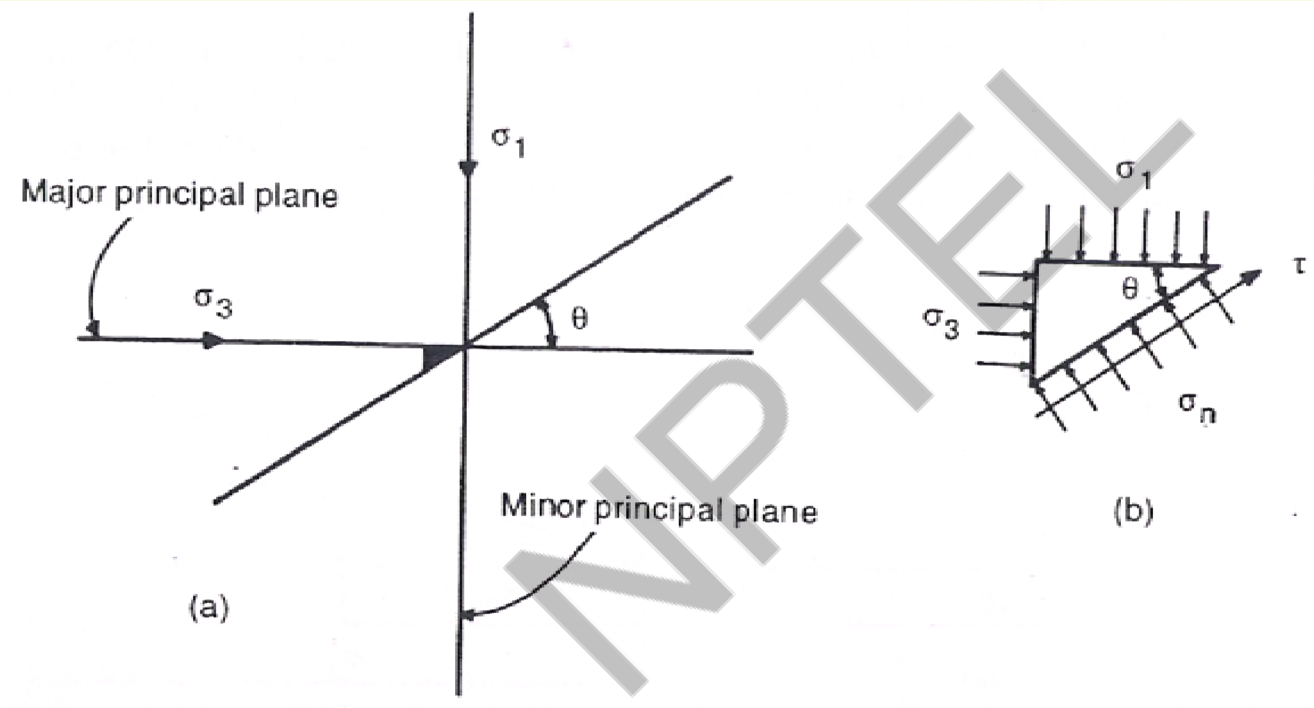
SHEAR STRENGTH



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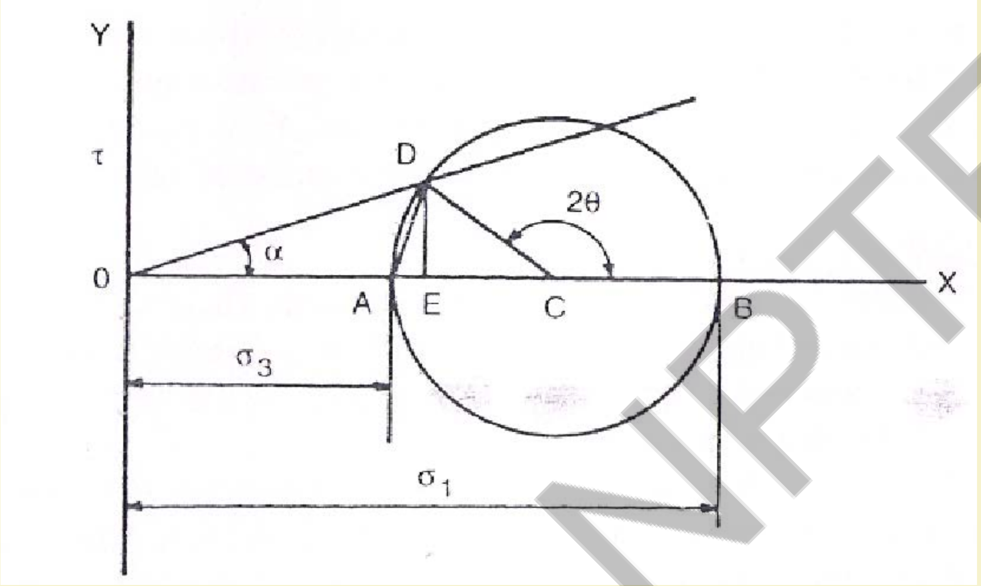

$$\tau = \frac{\sigma_1 - \sigma_3}{2}\ sin2\theta$$

$$\sigma_n = \sigma_3 + (\sigma_1 - \sigma_3)cos^2\theta$$

SHEAR STRENGTH

Limit conditions: It has been stated that the maximum shearing resistance is developed when the angle of obliquity equals its limiting value ϕ. For this condition line OD becomes a tangent to the stress circle, inclined at angle ϕ to axis OX.

An interesting point that arises from the Figure is that the failure plane is not the plane subjected to the maximum value of the shear stress. The criterion of failure is the maximum obliquity, not the maximum shear stress

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SHEAR STRENGTH

Strength envelopes: If ϕ is assumed constant for a certain material, then the shear strength of the material can be represented by a pair of lines passing through the origin , O at angles + ϕ and - ϕ to the axis OX. These lines comprise the Mohr strength envelope for the material

State of stress represented by Circle A is quite stable as the circle lies completely within the strength envelope, Circle B is tangential to the strength envelope and represent the condition of incipient failure. Circle C cannot exist as it is beyond the strength envelope.

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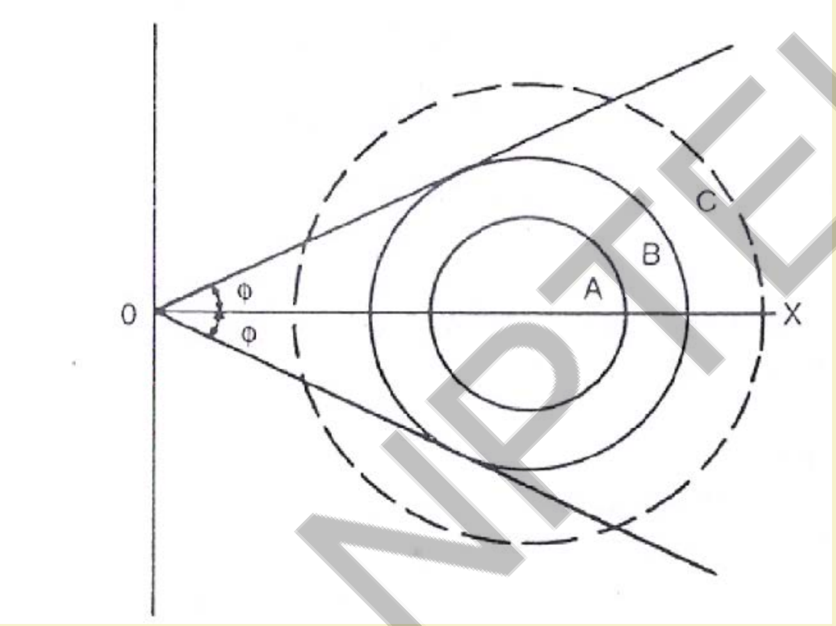
SHEAR STRENGTH



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Relationship between θ and ϕ : From the figure angle DCO = 180 -2 θ

In triangle ODC: Angle DOC = ϕ ODC = 90 deg OCD = 180 -2 θ and these angles summed to 180 deg,

$$\phi + 90 + 180 - 2\theta = 180$$

Hence, $\theta = 45 + \phi/2$

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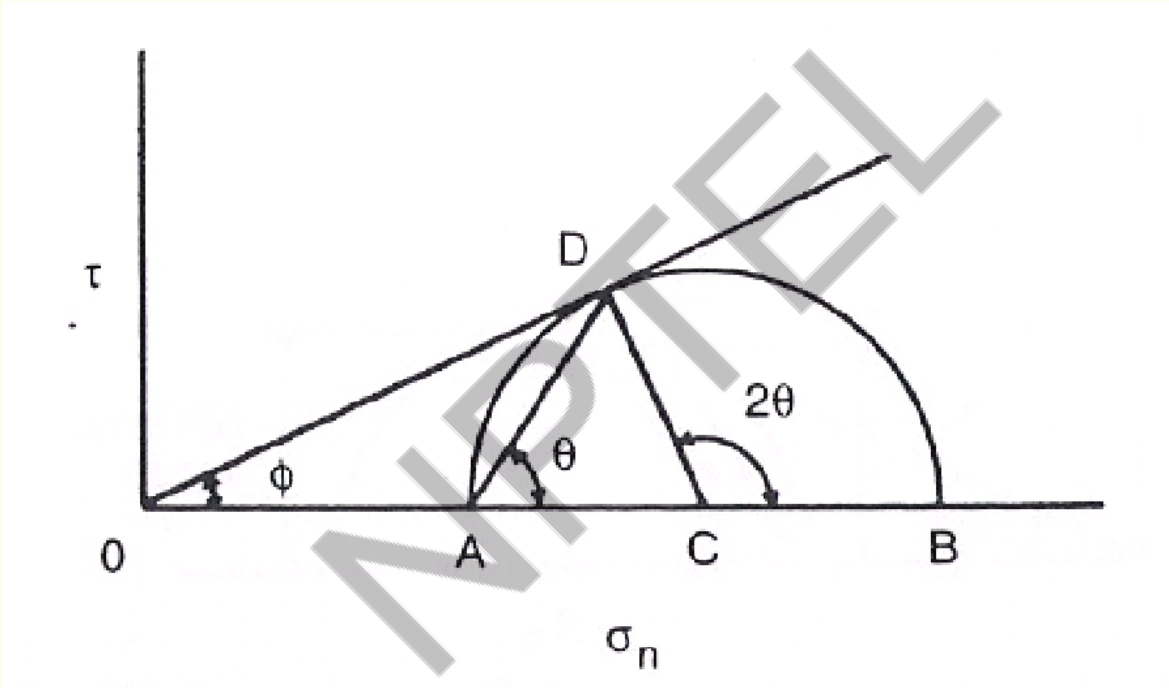
SHEAR STRENGTH



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Cohesion

It is possible to make a vertical cut in silts and clays and for this cut to remain standing, unsupported, for some time. This can not be done with dry sand which on removal of the cutting implement, will slump until its slope is equal to an angle known as angle of repose. In silts and clays, therefore some other factor must contribute to shear strength. This factor is called cohesion.

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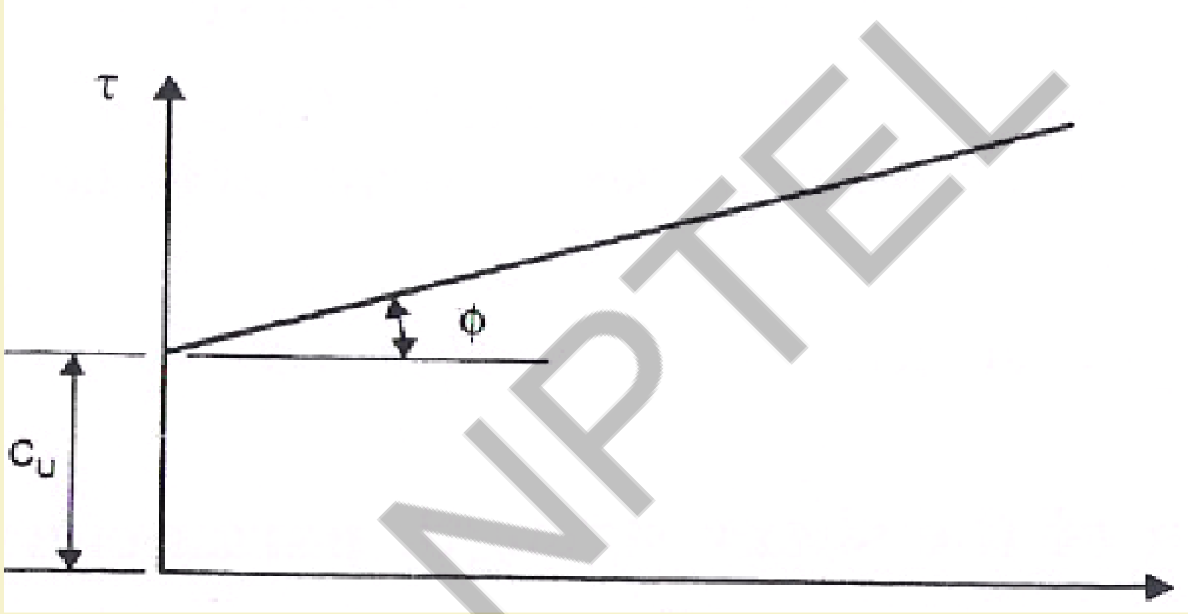
SHEAR STRENGTH



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SHEAR STRENGTH

It can be seen that the shear resistance offered by a particular soil is made of the two components of friction and cohesion. Frictional resistance does not have a constant value but varies with the value of normal stress acting on the shear plane whereas cohesive resistance has a constant value which is independent of normal stress.

$$\tau_f = c + \sigma \tan\varphi$$

τ_f shear strength at failure, C = cohesion, σ = normal stress

ø = angle of shearing resistance or angle of internal friction

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Shear strength depends upon effective stress and not on total stress. Coulomb's equation must therefore be modified in terms of effective stress and becomes:

$$\tau_f = c' + \sigma' \tan \varphi'$$

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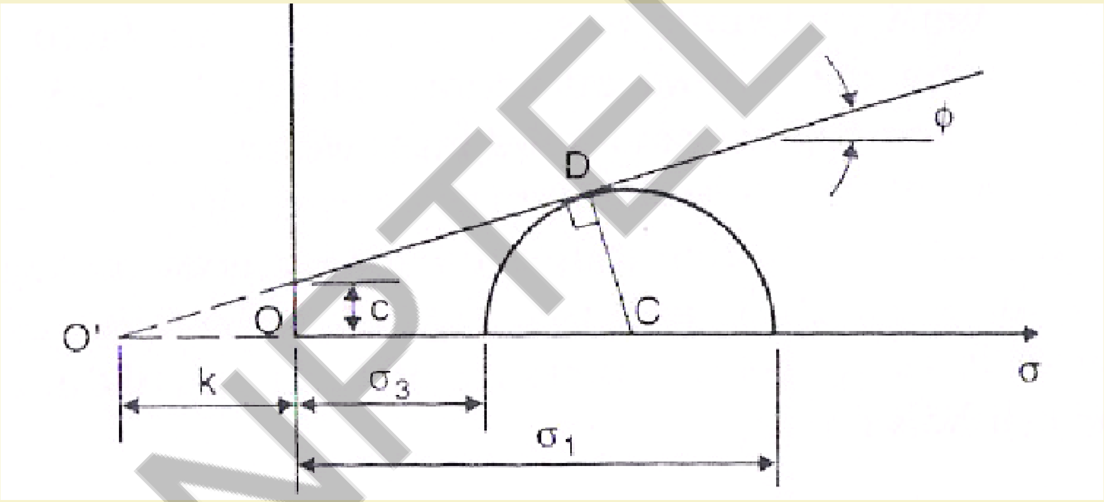
SHEAR STRENGTH



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$$\sin\emptyset = \frac{DC}{O'C} = \frac{\frac{1}{2}(\sigma_1 - \sigma_3)}{k + \frac{1}{2}(\sigma_1 + \sigma_3)} = \frac{(\sigma_1 - \sigma_3)}{2k + (\sigma_1 + \sigma_3)}$$

$$(\sigma_1 - \sigma_3) = 2k\sin\emptyset + (\sigma_1 + \sigma_3)\sin\emptyset$$

$$k = c\cot\emptyset$$

$$\sigma_1 - \sigma_3 = 2c\cos\emptyset + (\sigma_1 + \sigma_3)\sin\emptyset$$

The equation can be expressed in terms of either total stress as given above or effective stress as given below:

$$\sigma_1' - \sigma_3' = 2c\cos\emptyset' + (\sigma_1' + \sigma_3')\sin\emptyset'$$

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Thank You!!