

# Reinforced Concrete Road Bridges

Prof. Nirjhar Dhang  
Department of Civil Engineering  
Indian Institute of Technology  
Kharagpur

## Lecture-6

# Overview

- 1 Design of reinforced concrete elements
- 2 Working Stress Method
- 3 Working Stress Method : Shear
- 4 Summary
- 5 References

# Design of reinforced concrete elements

# Design of reinforced concrete elements

- In any structure, stresses may be developed due to
  - Axial force
  - Flexure
  - Shear
  - Torsion

# Design of reinforced concrete elements

- The superstructure of the bridge is mainly governed by
  - Flexure
  - Shear

# Design of reinforced concrete elements

- Therefore, design principles will be discussed on
  - Flexure
  - Shear

# Design of reinforced concrete elements

- Further, we shall consider the following codes for discussion
  - Working Stress Method (as per IRC 21:2000 and IS 456:2000)
  - Limit State Method (as per IS 456:2000)
  - Limit State Method (as per IRC 112:2011)

# Working Stress Method



# Working Stress Method

## Assumptions

- At any cross-section, plane sections before bending remain plain after bending
- All tensile stresses are taken up by reinforcement and none by concrete, except as otherwise specified
- The stress-strain relationship of steel and concrete, under working loads, is a straight line
- The modular ratio  $m$  has the value  $\frac{280}{3\sigma_{cbc}}$

# Working Stress Method

Permissible stresses  $N/mm^2$  in concrete

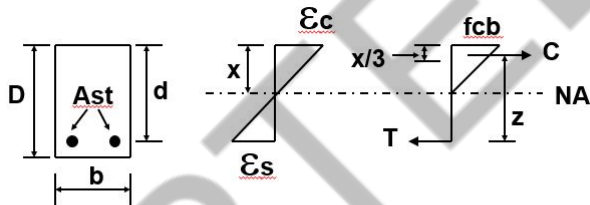
Grade of Concrete	Bending compression (as per IRC 21:2000)	Direct compression (as per IRC 21:2000)	Bond (average) Plain bars as per (IS 456:2000)
M25	8.33	6.25	0.9
M30	10.0	7.5	1.0
M35	11.67	8.75	1.1
M40	13.33	10	1.2

# Working Stress Method

Permissible stresses  $N/mm^2$  in steel  
(Table 10 : IRC 21:2000)

Type of steel	Tension bending	Compression	Shear
Fe240	125	115	125
Fe415	200	170	200
Fe500	240	205	240

# Working Stress Method



$$x = k d$$

$$z = j d$$

$$j = 1 - k / 3$$

$$m = 280 / (3 \sigma_{cbc})$$

$$C = (1/2) f_{cb} b x$$

$$T = f_{st} A_{st}$$

$$M = C z = T z$$

# Working Stress Method

$$x = kd \quad m = \frac{280}{3\sigma_{cbc}}$$

$$Z = jd \quad C = \frac{1}{2}f_{cb}bx$$

$$j = 1 - \frac{k}{3} \quad T = f_{st}A_{st}$$

$$M = Cz = Tz$$

(1)

# Working Stress Method

$$M = \frac{1}{2} \sigma_{cbc} (kd) (bd) \left(1 - \frac{k}{3}\right)$$

$$M = \frac{1}{2} \sigma_{cbc} k \left(1 - \frac{k}{3}\right) bd^2$$

$$k_b = \frac{m \sigma_{cbc}}{\sigma_{st} + m \sigma_{cbc}}$$

(2)

$$m = \frac{280}{3 \sigma_{cbc}}$$

$$b(kd) \frac{kd}{2} = \frac{p}{100} (bd) md (1 - k)$$

# Working Stress Method

**Balanced sections:** Sections, in which the tension steel reaches yield strain simultaneously as the concrete reaches the failure strain in bending are called balanced sections

# Working Stress Method

**Under reinforced sections:** Sections, in which tension steel reaches yield strain at load slower than the load at which concrete reaches failure strain, are called under reinforced sections



# Working Stress Method

**Overreinforced sections:** Sections, in which the failure strain in concrete is reached earlier than the yield strain of steel is reached, are called overreinforced sections

# Working Stress Method

**Balanced sections:** Sections, in which the tension steel reaches yield strain simultaneously as the concrete reaches the failure strain in bending are called balanced sections

**Under reinforced sections:** Sections, in which tension steel reaches yield strain at load slower than the load at which concrete reaches failure strain, are called under reinforced sections

**Overreinforced sections:** Sections, in which the failure strain in concrete is reached earlier than the yield strain of steel is reached, are called overreinforced sections

## Working Stress Method : Shear

# Working Stress Method : Shear

- Shear force,  $V$
- Shear force to be taken by  $V_c = \tau_c bd$
- Shear force to be resisted by stirrups,  $V_s = V - V_c$

# Working Stress Method : Shear

- Shear reinforcement shall be provided to carry a shear  $V_s = V - \tau_c bd$  to be calculated as follows:

$$A_{sv} = \frac{V_s s}{\sigma_s d (\sin \alpha + \cos \alpha)} \quad (3)$$

# Working Stress Method : Shear

- Shear reinforcement shall be provided to carry a shear  $V_s = V - \tau_c bd$  to be calculated as follows:

$$A_{sv} = \frac{V_s s}{\sigma_s d (\sin \alpha + \cos \alpha)} \quad (4)$$

- $A_{sv}$  = total cross-sectional area of stirrups or bent-up bars within a distance  $s$ .
- $s$  = spacing of the stirrups or bent-up bars along the length of the member
- $b$  = breadth of the member which for flanged beams, shall be taken as the breadth of the web
- $\alpha$  = angle between the inclined stirrup or bent up bar and the axis of the member, not less than  $45^\circ$
- $\sigma_s$  = permissible stress in shear reinforcement
- $d$  = the effective depth

# Working Stress Method : Shear

Permissible shear strength in concrete

$$\tau_c \text{ in } N/mm^2$$

(Part of Table 12B, IRC 21:2000)

$100 \frac{A_s}{bd}$	Grade of concrete				
	M20	M25	M30	M35	M40
0.15	0.18	0.19	0.2	0.2	0.2
0.25	0.22	0.23	0.23	0.23	0.23
0.50	0.3	0.31	0.31	0.31	0.32
0.75	0.35	0.36	0.37	0.37	0.38
1.00	0.39	0.4	0.41	0.42	0.42
1.25	0.45	0.46	0.48	0.49	0.49
1.50	0.47	0.49	0.5	0.52	0.52
1.75	0.49	0.51	0.53	0.54	0.55
2.00	0.51	0.53	0.55	0.56	0.57

# Summary



# Summary

- Working stress method for design in bending and shear are discussed.

## References

# References

- IRC 21 : 2000** Standard specifications and code of practice for road bridges, Section III : Cement concrete (plain and reinforced) (Indian Roads Congress, New Delhi)
- IRC 112 : 2011** Code of practice for concrete road bridges (Indian Roads Congress, New Delhi)
- IS 456 : 2000** Indian Standard Plain and Reinforced Concrete (Bureau of Indian Standards, New Delhi)

Thank you

# Reinforced Concrete Road Bridges

Prof. Nirjhar Dhang

Department of Civil Engineering

Indian Institute of Technology

Kharagpur

Lecture-7

# **Limit State Method**

**IS 456 : 2000**

# **Limit State of Collapse**

## **Flexure**

# Assumptions

- Plane sections normal to the axis remain plane after bending
- The maximum strain in concrete at the outermost compression fibre is taken as 0.0035
- The tensile strength of the concrete is ignored



# Assumptions (contd..)

- The stress block may be assumed to be rectangle, trapezoid, parabola or any other shape which results in prediction of strength in substantial agreement with the results of test

**Note :** *Triangular as in the Working Stress Method*

## Assumptions (contd..)

- The stresses in the reinforcement are derived from representative stress-strain curve for the type of steel used.

For design purposes, the partial safety factor  $\gamma_m$ , equal to 1.15 shall be applied.

# Assumptions (contd..)

- The maximum strain in the tension reinforcement in the section at failure shall not be less than:

$$\frac{f_y}{1.15E_s} + 0.002$$

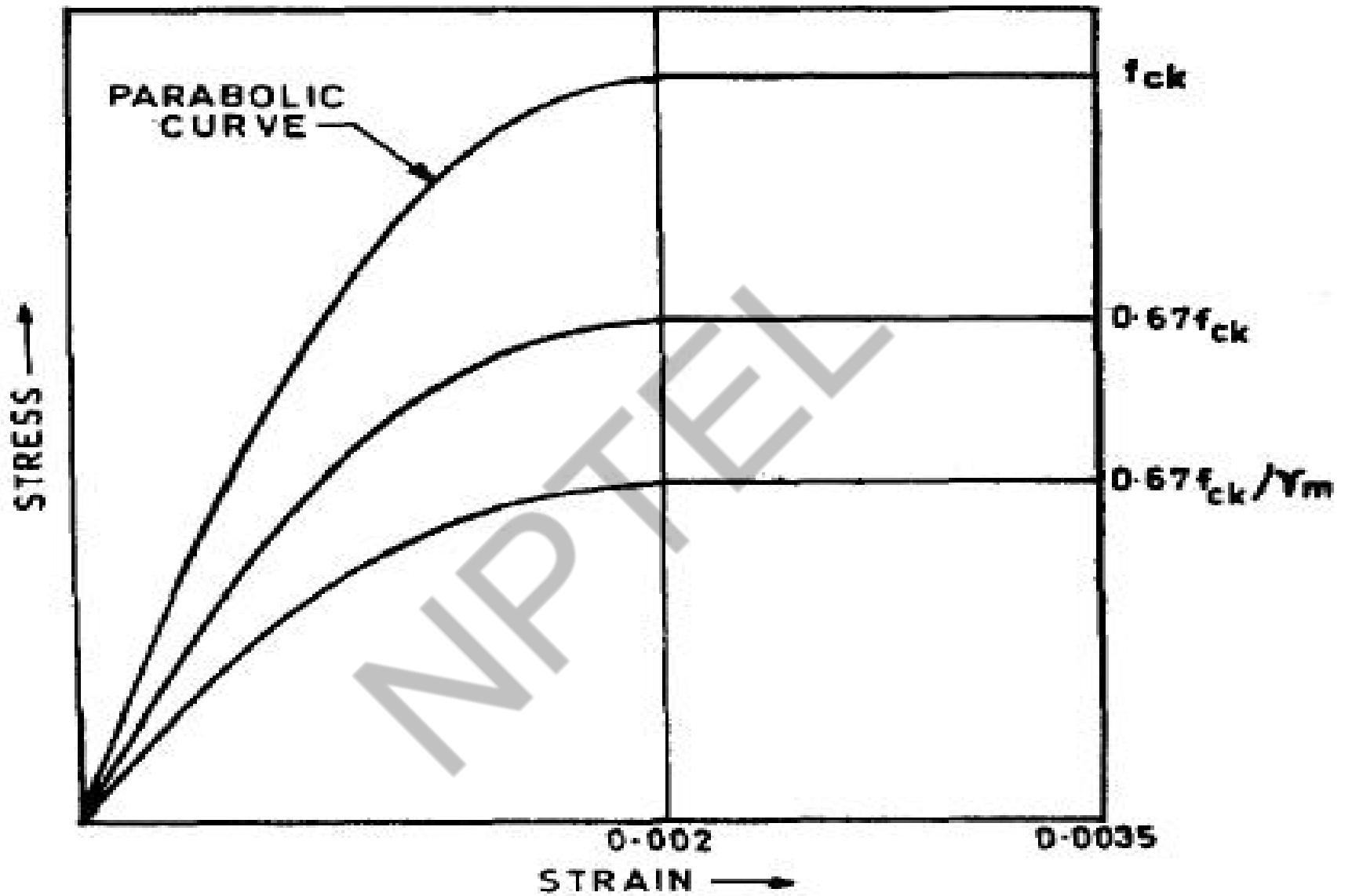
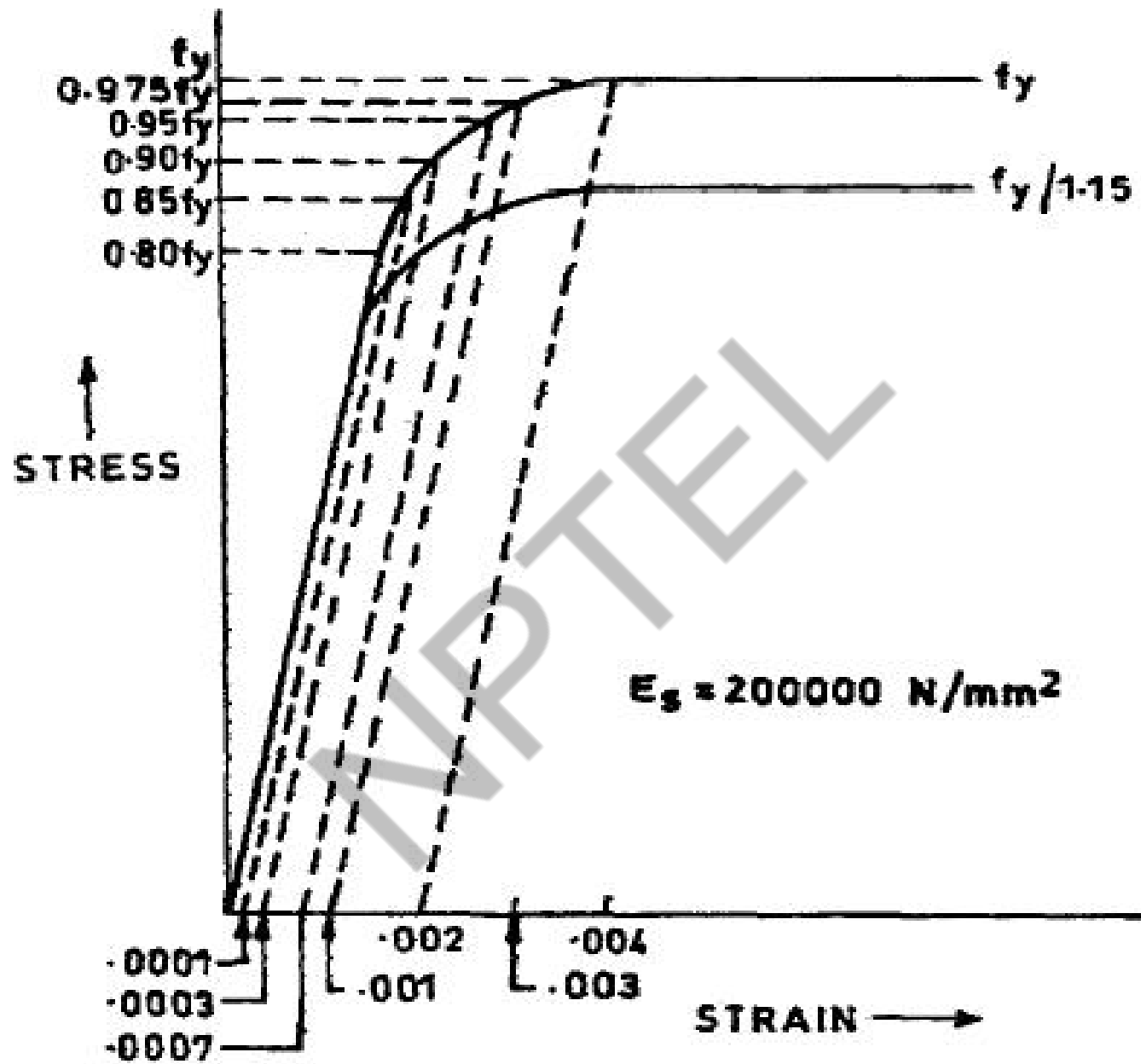
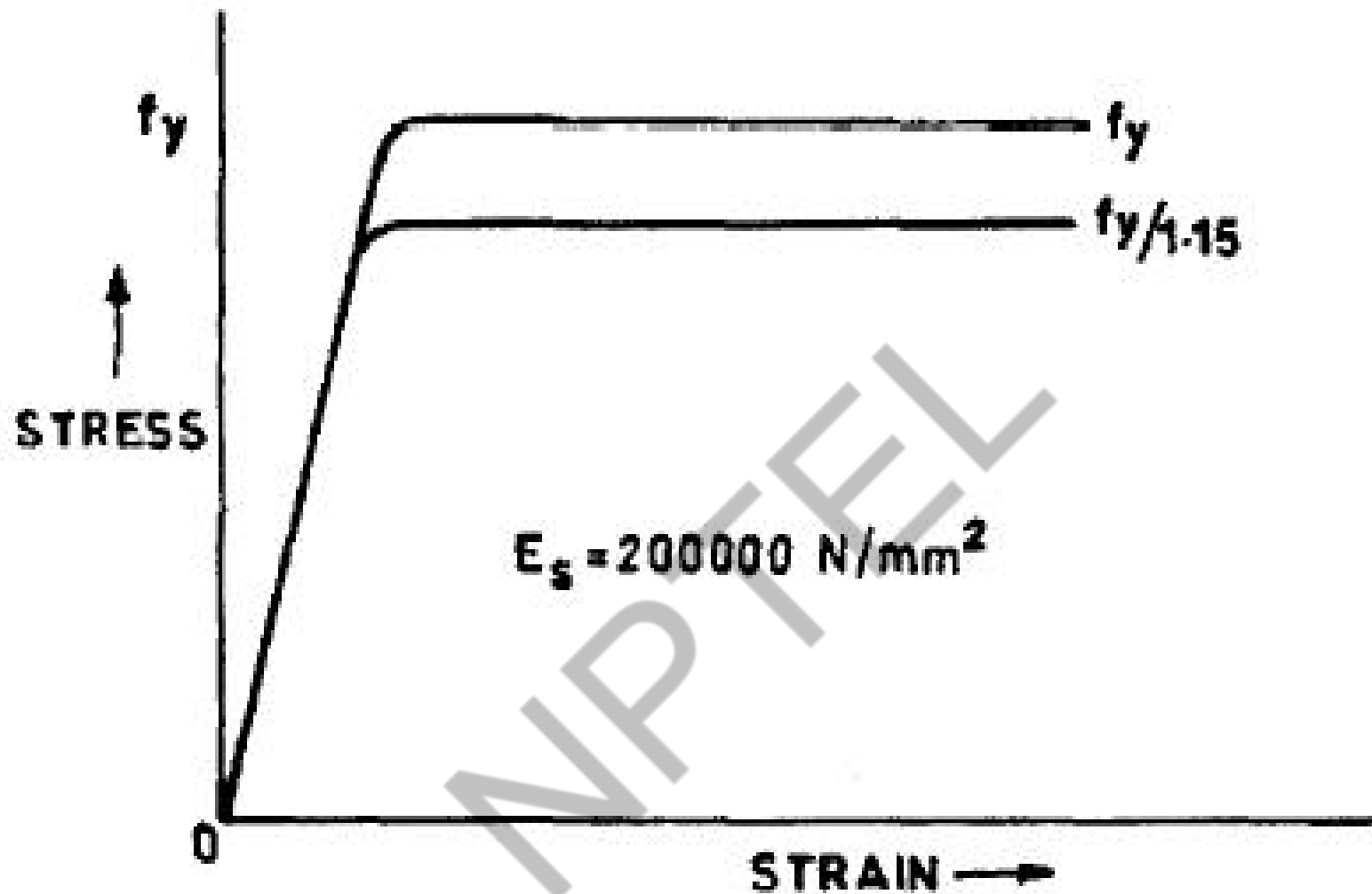


FIG. 21 STRESS-STRAIN CURVE FOR CONCRETE

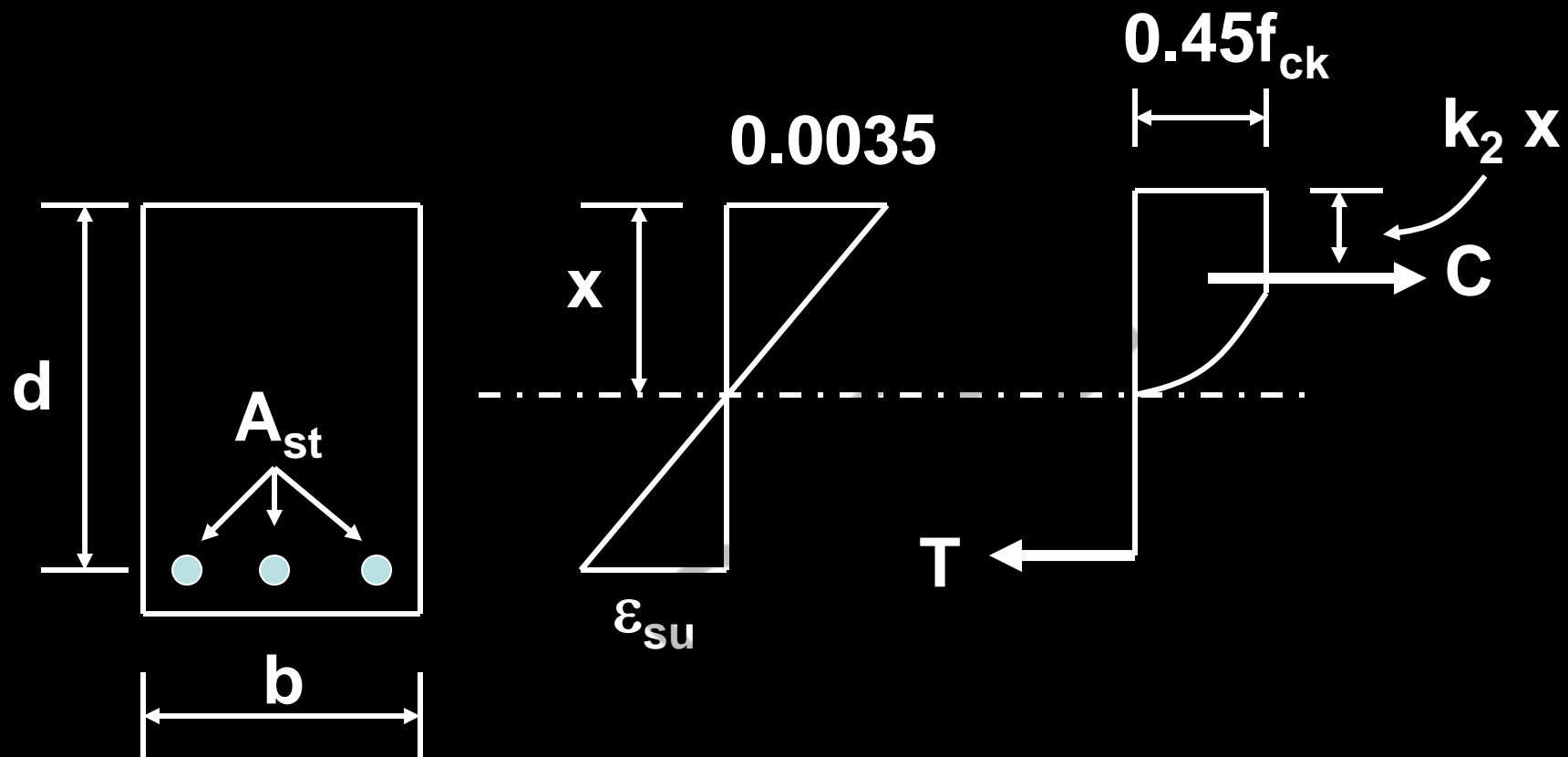


23A Cold Worked Deformed Bar



23B STEEL BAR WITH DEFINITE YIELD POINT

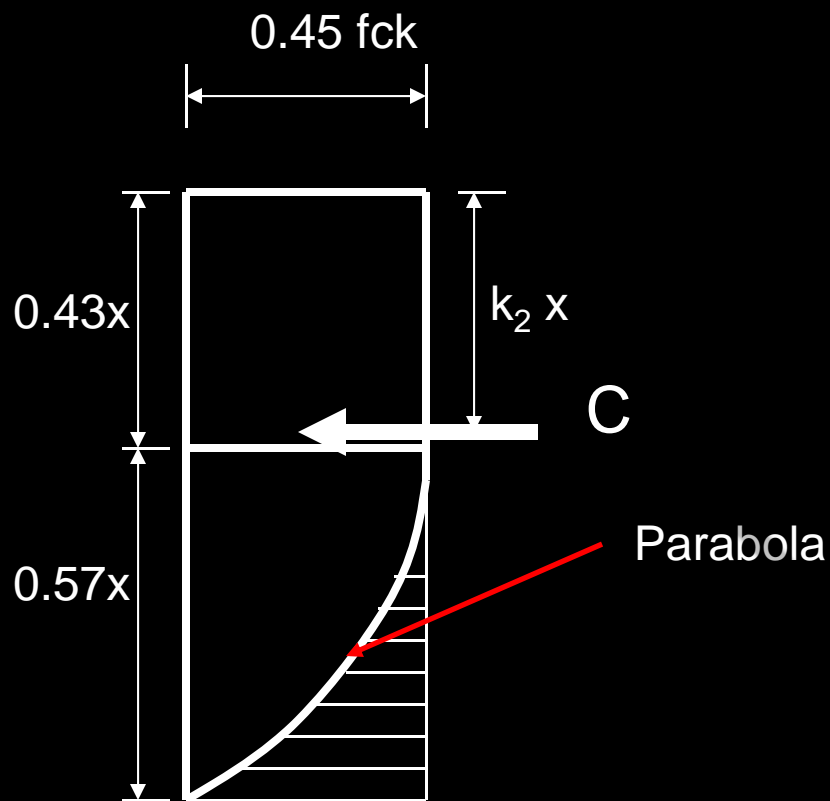
FIG. 23 REPRESENTATIVE STRESS-STRAIN CURVES FOR REINFORCEMENT



**$k_2x$  is the distance of the  
centre of compression in the  
concrete from the top  
compression fibre**



# Determination of constants $k_1$ and $k_2$ for compression stress block



$C$  = area of rectangle –  
area of outer parabola

$$= 0.45 f_{ck} (x) b - 0.45 f_{ck} (0.57/3) (x) b$$

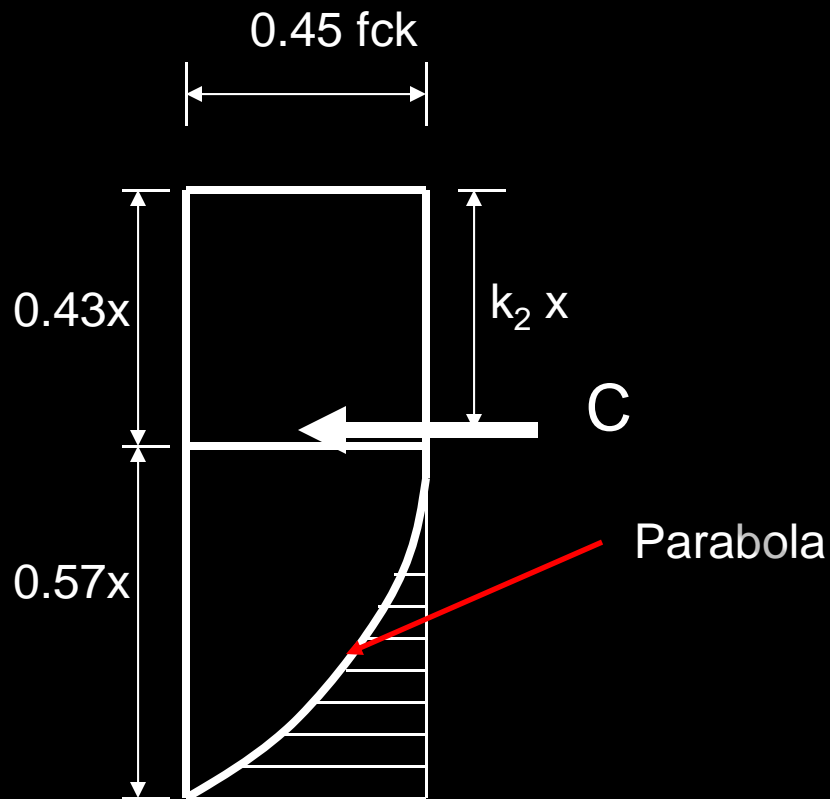
$$= 0.45 f_{ck} (x) b - 0.086 f_{ck} (x) b$$

$$= 0.364 f_{ck} (x) b$$

Therefore,

$$k_1 = 0.364$$

## Determination of constants $k_1$ and $k_2$ for compression stress block



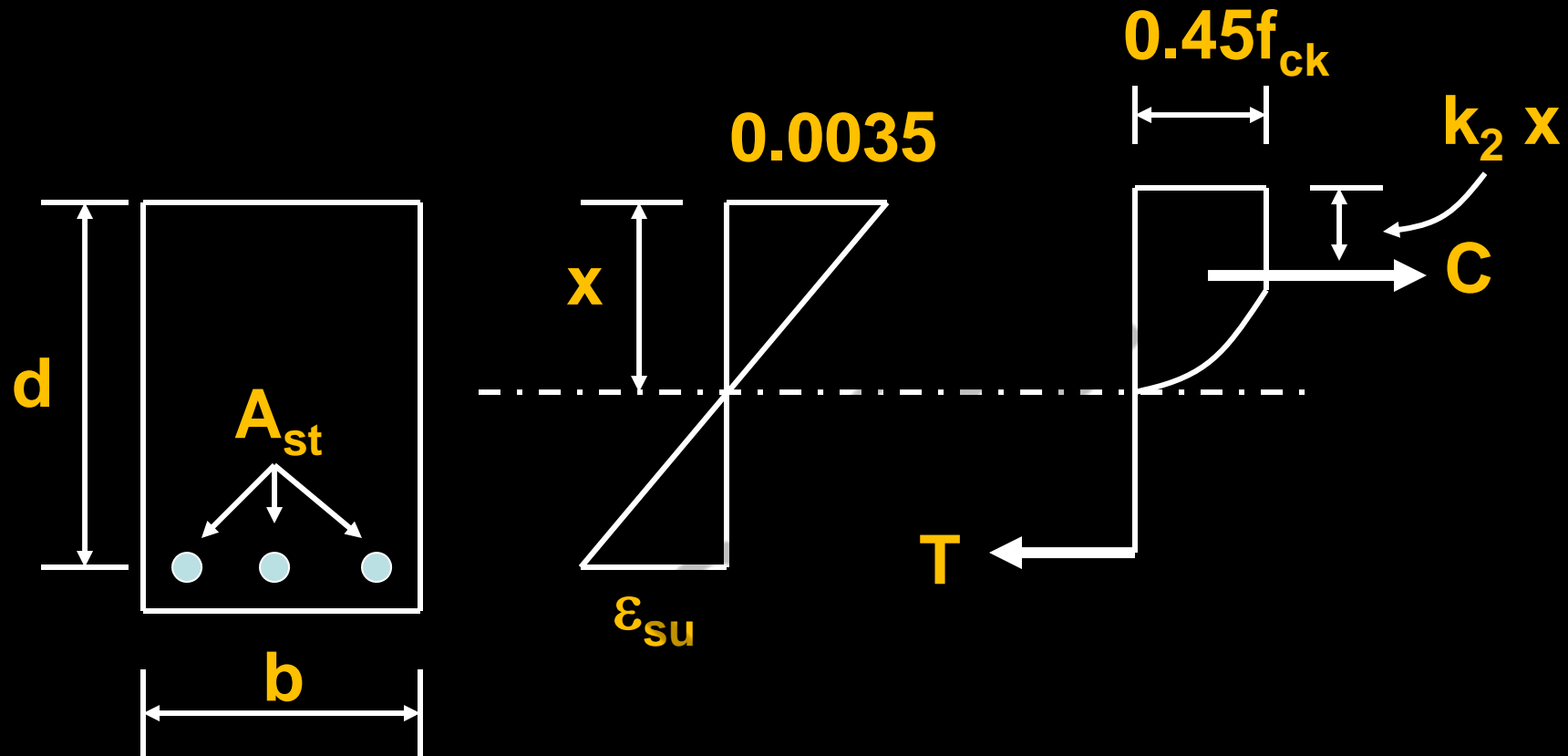
$$C = 0.364 f_{ck} (x) b$$

Taking moment about NA

$$[0.364 f_{ck} (x) b] [x - k_2 x] = 0.45 f_{ck} (x) b (x/2) - 0.45 f_{ck} (0.57x)^2 b / 12$$

$$1 - k_2 = 0.584$$

$$k_2 = 0.416 = 0.42$$



$$C = k_1 f_{ck} (x/d) bd$$

$$T = 0.87 f_y A_{st}$$

$$C = k_1 f_{ck} (x/d) b d$$

$$T = 0.87 f_y A_{st}$$

$$0.87 f_y A_{st} = 0.364 f_{ck} b x_u$$

$$M_u = k_1 f_{ck} \left( \frac{x}{d} \right) \left( 1 - k_2 \left( \frac{x}{d} \right) \right) b d^2$$

$$j = 0.5 + \sqrt{0.25 - \frac{M_u}{0.87 f_{ck} b d^2}}$$

# **LIMITING VALUES OF $x/d$**

<b>Type of steel</b>	<b><math>f_y</math></b>	<b>Yield strain (<math>\epsilon_{su}</math>)</b>	<b><math>X_u/d</math></b>
<b>Mild steel</b>	<b>250</b>	<b>0.0031</b>	<b>0.53</b>
<b>High yield strength</b>	<b>415</b>	<b>0.0038</b>	<b>0.48</b>
<b>High yield strength</b>	<b>500</b>	<b>0.0042</b>	<b>0.46</b>

# PERCENTAGE OF LIMITING STEEL AREAS FOR BALANCED DESIGN

Steel	$x/d$	$p_t(f_y/f_{ck})$
Fe250	0.53	21.97
Fe415	0.48	19.82
Fe500	0.46	18.87

**The maximum or limiting value of  
compression,**

$$C_L = F f_{ck} bd$$

# VALUES OF CONSTANTS FOR MAXIMUM COMPRESSION BLOCK

Steel	$X_u/d$	$k_1$	$k_2$	F
Fe250	0.53	0.364	0.42	0.192
Fe415	0.48	0.364	0.42	0.175
Fe500	0.46	0.364	0.42	0.167



# Balanced Sections

**Sections, in which the tension steel reaches yield strain simultaneously as the concrete reaches the failure strain in bending are called balanced sections**

# **Underreinforced sections**

**Sections, in which tension steel reaches yield strain at loads lower than the load at which concrete reaches failure strain, are called under reinforced sections**

# **Overreinforced sections**

**Sections, in which the failure strain in concrete is reached earlier than the yield strain of steel is reached, are called overreinforced sections**

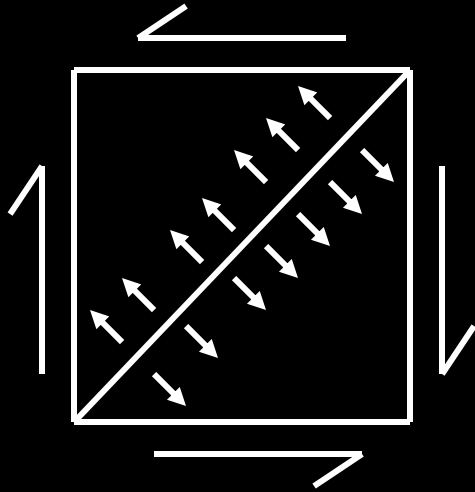
# PERCENTAGE OF LIMITING STEEL AREAS FOR BALANCED DESIGN

Steel	$x/d$	$p_t(f_y/f_{ck})$
Fe250	0.53	21.97
Fe415	0.48	19.82
Fe500	0.46	18.87

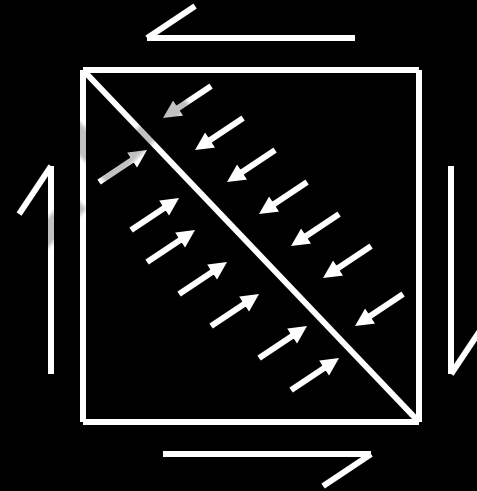
# **Limit State of Collapse**

## **Shear**

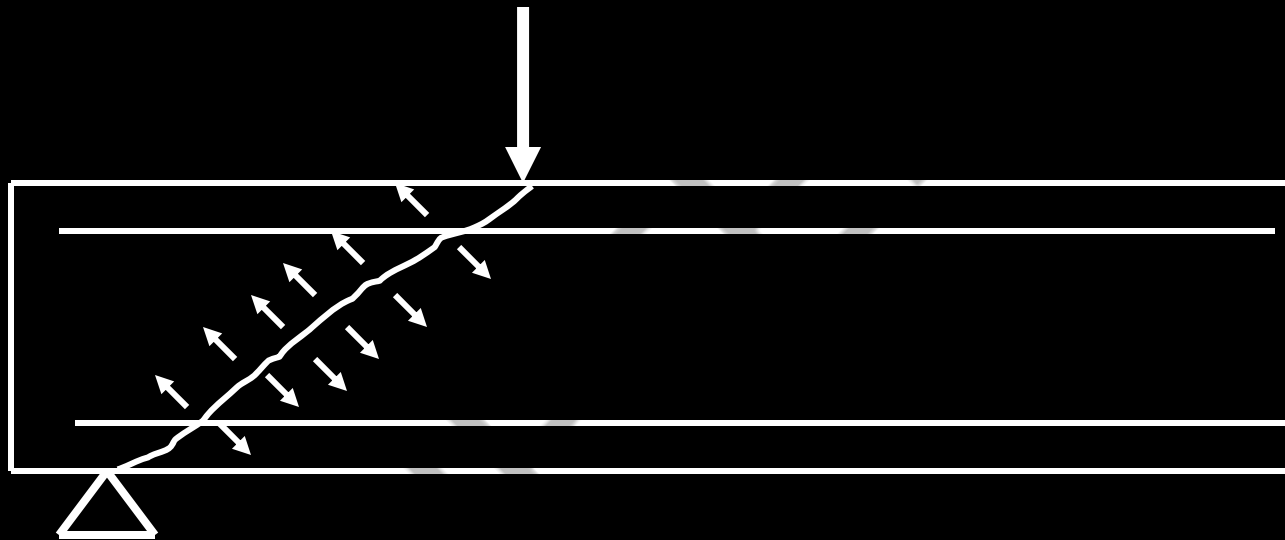
## Shear and diagonal stresses



**(a) Diagonal tension**



**(b) Diagonal compression**



**Tension in Beams**

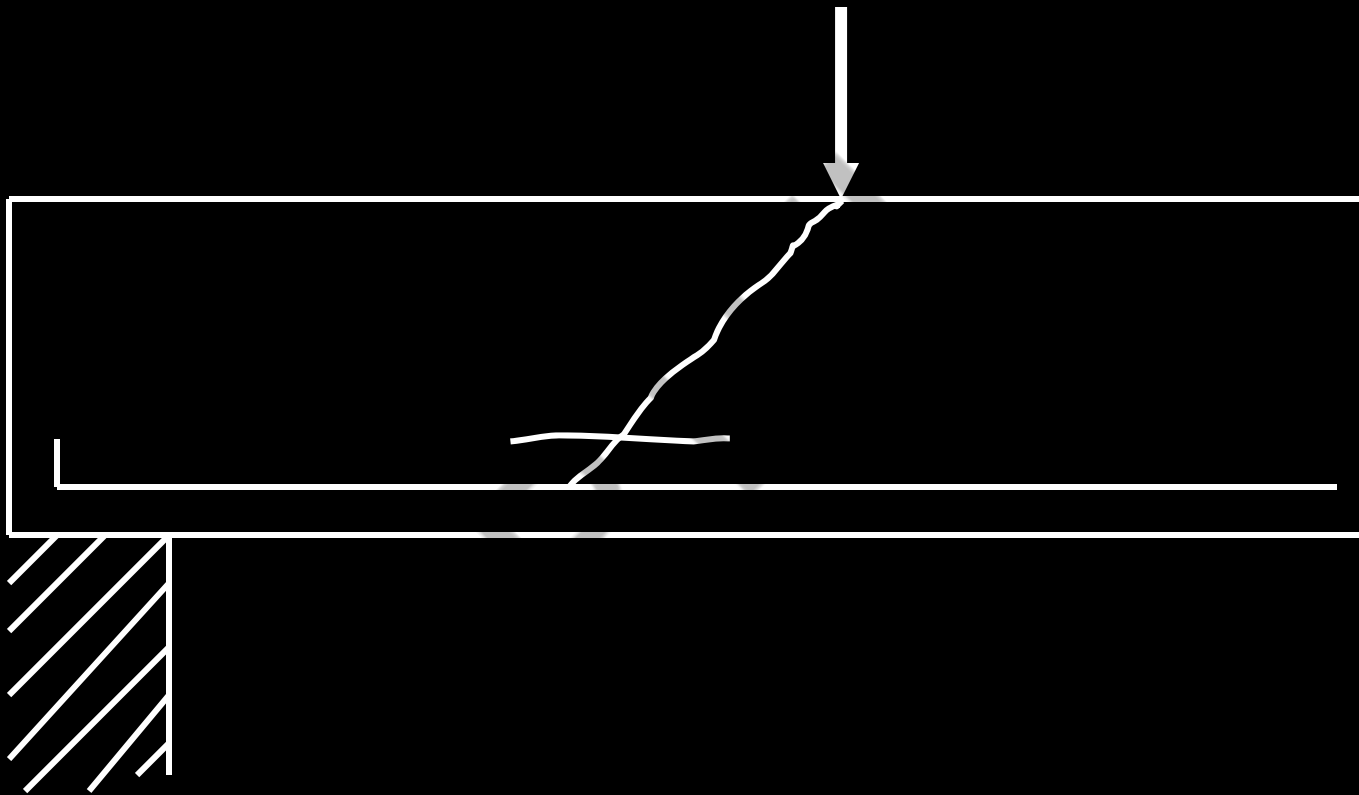


**Shear failure : Shear tension**

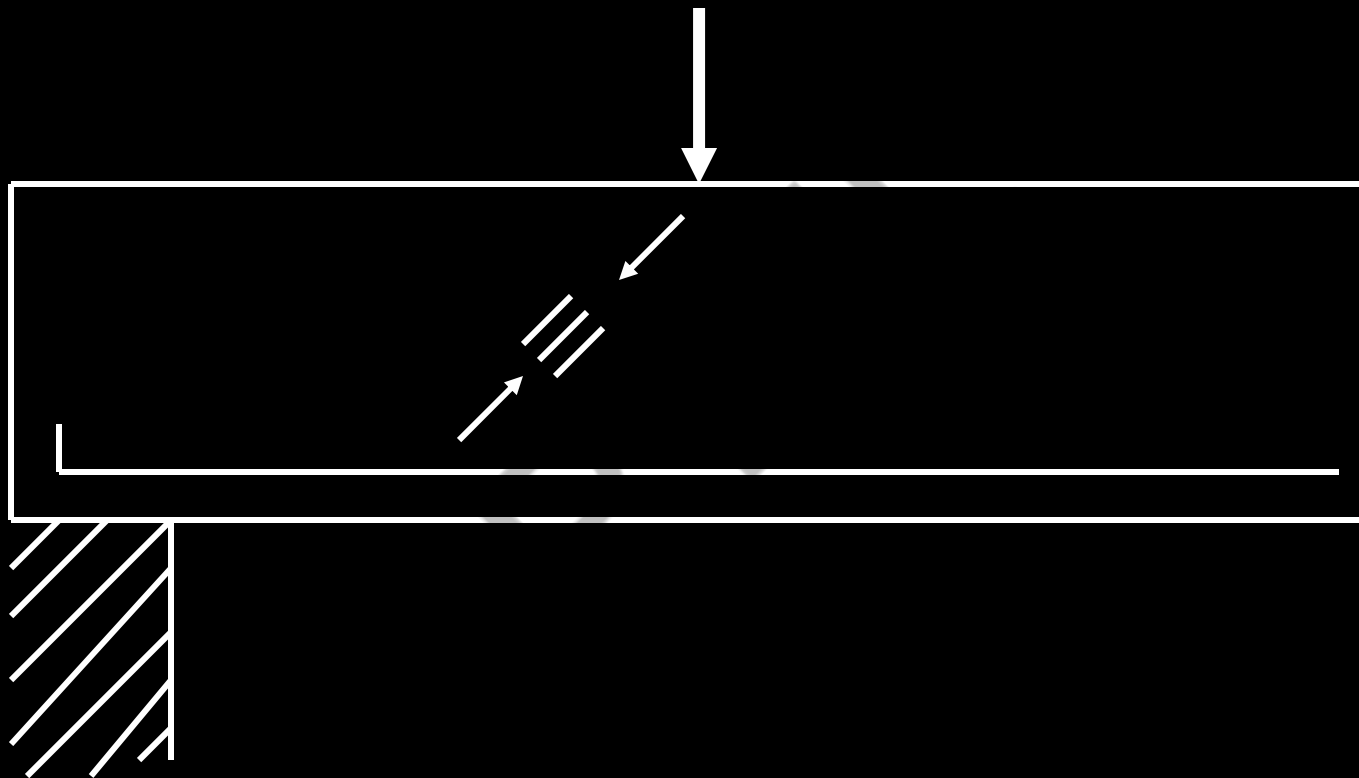




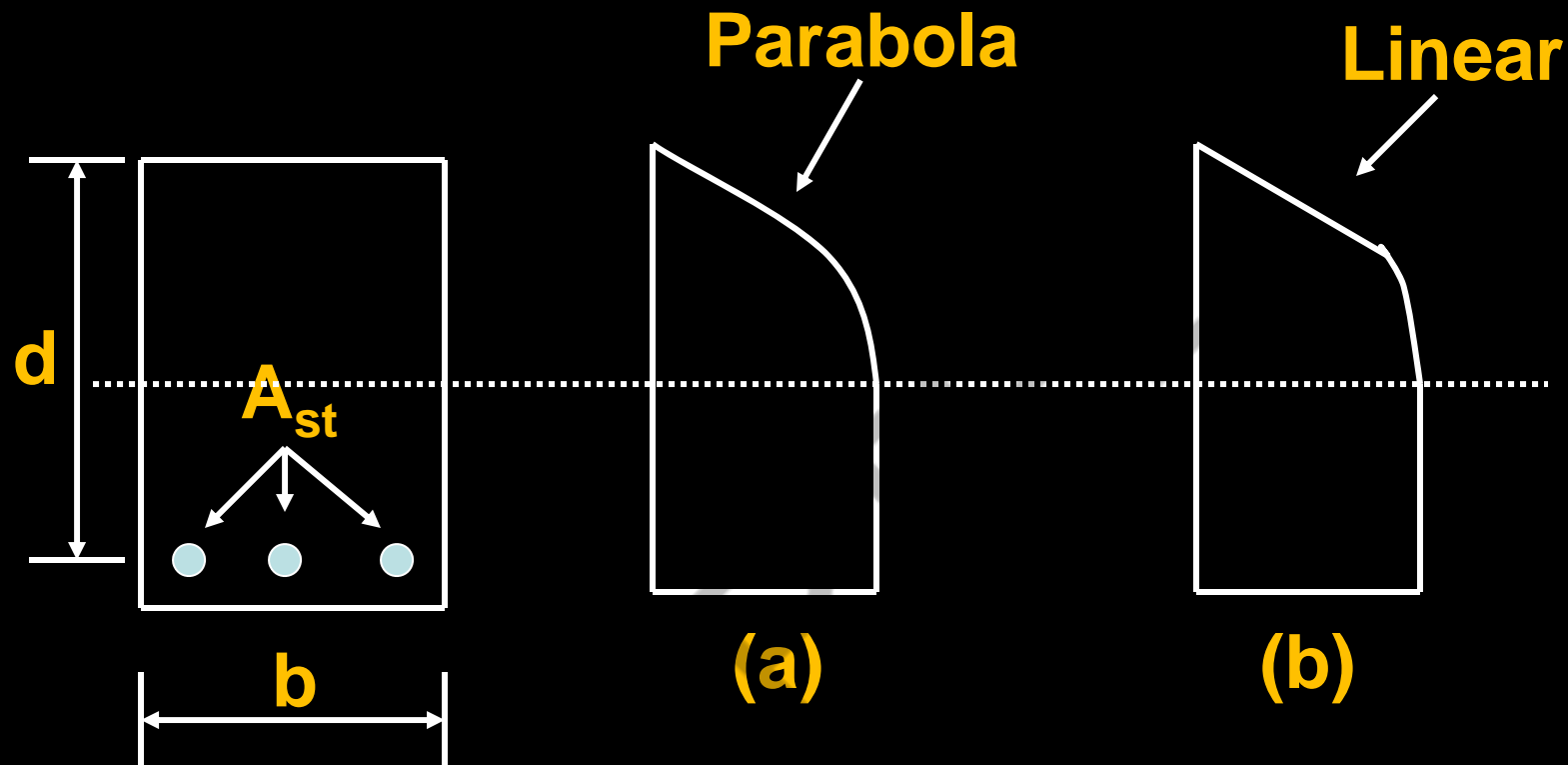
**Shear failure : Shear bending**



**Shear failure : Shear bond**



**Shear failure : Shear compression**



## Shear stress distribution in RCC beams

(a) Elastic stage      (b) Ultimate stage

# Nominal shear stress

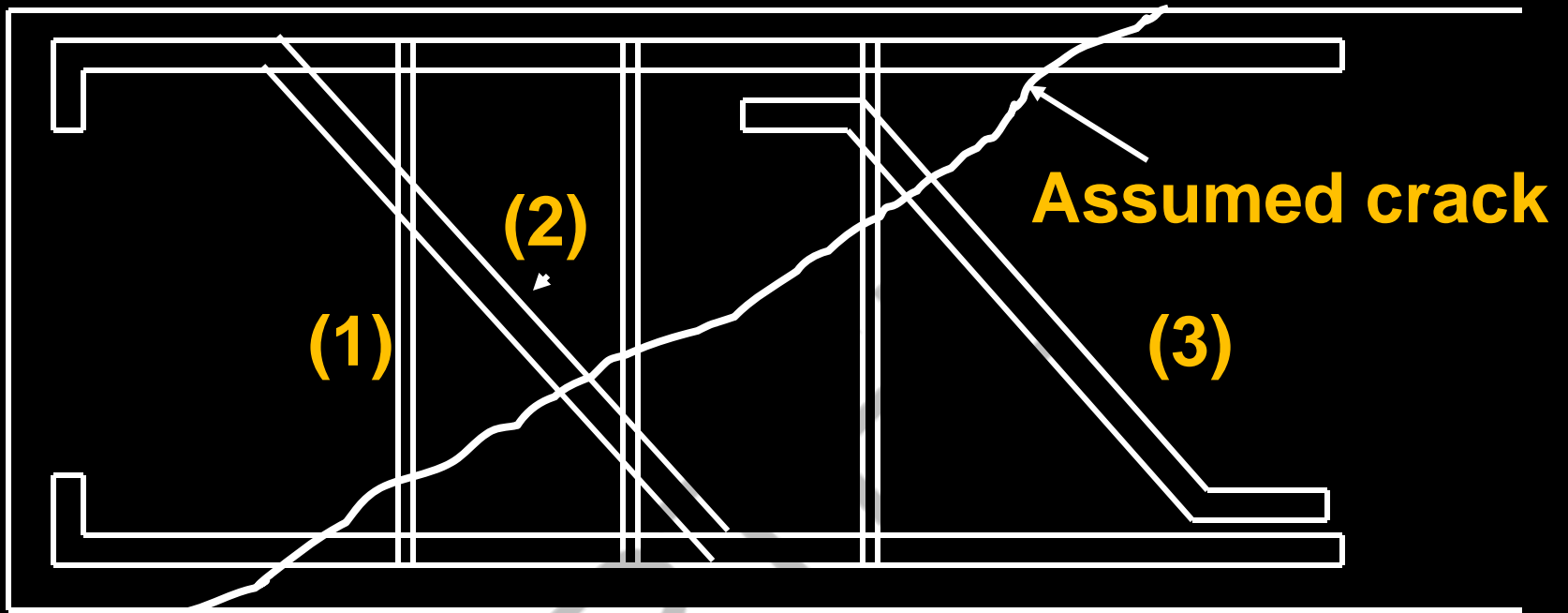
The nominal shear stress in beams of uniform depth shall be obtained by

$$\tau_v = \frac{V_u}{bd}$$

**$V_u$  = shear force due to design loads**

**$b$  = breadth of the member, which for flanged section shall be taken as the breadth of the web**

**$d$  = effective depth**



- (1) Bent-up tension bar**
- (2) System of vertical stirrups**
- (3) System of inclined stirrups**

**The shear forces are resisted  
by combined action of  
concrete and shear steel**



## **The resistance in concrete is due to the combined effect of three factors**

- **$V_{cz}$ , shear carried by uncracked section ( 20 to 40 of total shear in compression**
- **$V_a$ , shear carried by aggregate interlock across the diagonal crack (33 to 50 %)**
- **$V_d$ , shear carried by dowel action of longitudinal tension steel (15 to 25 %)**

# Shear carried by concrete

$$V_c = V_{cz} + V_a + V_d$$

# Total shear to be carried

$$V = V_{\text{concrete}} + V_{\text{stirrup}}$$

or

$$V = V_c + V_s$$

**Part of Table 19**  
**Design Shear Strength of Concrete,  $\tau_c$ , N/mm<sup>2</sup>**

$100A_s$ /bd	M20	M25	M30	M35	M40
0.15	0.28	0.29	0.29	0.29	0.30
0.25	0.36	0.36	0.37	0.37	0.38
0.50	0.48	0.49	0.50	0.50	0.51
0.75	0.56	0.57	0.59	0.59	0.60
1.00	0.62	0.64	0.66	0.67	0.68
1.25	0.67	0.70	0.71	0.73	0.74

# Table 20 Maximum Shear Stress, $\tau_{cmax}, \text{N/mm}^2$

(Clauses 40.2.3, 40.2.3.1, 40.5.1 and 41.3.1)

Grade Of concr ete	M15	M20	M25	M30	M35	M40 and above
$\tau_{cmax},$  N/ mm <sup>2</sup>	2.5	2.8	3.1	3.5	3.7	4.0

- Shall not exceed  $0.75 d$  for vertical stirrups
- Shall not exceed  $d$  for inclined stirrups at 45 degree

Where  $d$  is the effective depth under consideration

In no case shall the spacing exceed 300 mm

Thank you

# Reinforced Concrete Road Bridges

Prof. Nirjhar Dhang  
Department of Civil Engineering  
Indian Institute of Technology  
Kharagpur

## Lecture-8



# Overview

- 1 Limit State Method (IRC 112:2011)
- 2 Limit State of Collapse : Flexure
- 3 Summary
- 4 References

# Limit State Method (IRC 112:2011)

# General performance requirements as per IRC 112:2011

- Clause 5.1.1 : The bridge, as a complete structural system and its structural elements should perform their functions adequately and safely, with appropriate degrees of reliability during design life and during construction.

# General performance requirements as per IRC 112:2011

- Clause 5.1.1 : It should withstand all actions, consisting of applied and induced loads as well as environmental influences liable to occur, retaining its structural integrity, and also withstand accidental loads (e.g. barge impact/vehicular impact) and earthquake loads without causing damage, which is disproportionate to the causative event.
- Adequacy of performance is defined in terms of serviceability, safety, durability and economy

# Reliability aspects and codal approach

- The term degree of reliability is used to indicate the acceptably low level of probability of failure in meeting the expected performance during a specified period of time.
- Determination of the reliability measured in terms of statistical probability requires knowledge of statistical parameters which define loading and material strengths.
- This data together with knowledge of structural models of resistance enable evaluation of structural performance in probabilistic terms.
- At the present state of knowledge, determination of reliability is possible only in limited load cases for simple structures.

# Reliability aspects and codal approach

- The Code, therefore, strives to achieve the desirable degree of reliability by approximate methods based upon a combination of the following:
  - Known statistical parameters describing properties of materials and actions.
  - Deterministic models of structural behaviour.
  - The international practices and past experience of acceptable / unacceptable performance of structures.
  - Partial factors for actions and resistance models based on calibration and rationalisation of existing international practices

# Limit State Philosophy of design

- The response of the structure when subjected to different magnitudes of loads lies in different states (domains). Limit States are defined as limits of domains beyond which the structure does not meet specified performance criteria.
- In Limit State Philosophy of design, various boundaries of acceptable/unacceptable performance are defined together with the circumstances in which such performances are expected.

# Limit State Philosophy of design

- Two basic groups of limit states are considered:
  - (a) Ultimate Limit States (ULS) : These limit states cover static equilibrium and failure of structural elements or structure as a whole, when acted upon by ultimate design loads
  - (b) Serviceability Limit States (SLS): These limit states deal with the condition of the structure subjected to influence of serviceability design loads.
  - These conditions include level of internal stress, fatigue failure, deflection, damage to structural element such as cracking, and discomfort to users due to vibrations.



# Limit State Philosophy of design

- The representative values of actions and combination of actions representing different design situations are defined.
- The representative values of loads are modified by using load factors for each of the basic limit states, which are then combined using combination factors.
- The combination factors take into account the probability of simultaneous occurrence of loads.

# Limit State Philosophy of design

- The response of the structure is calculated using principles of mechanics and simplified established models describing behaviour of concrete members.
- These methods also account for inherent geometric variations which are kept within acceptable construction tolerances.
- The response of the structure is required to lie within acceptable domain for different combinations of actions.
- The structure designed by following this philosophy, and constructed by satisfying other stipulations of the Code are deemed to meet the general performance requirements stipulated in Clause 5.1.1.

# Limit States

## Ultimate limit states (ULS)

- Limit state of equilibrium
  - When subjected to various design combinations of ultimate loads the bridge or any of its components considered as a rigid body, shall not become unstable.
- Limit state of strength
  - The bridge or any of its components shall not lose its capacity to sustain the various ultimate load combinations by excessive deformation, transformation into a mechanism, rupture, crushing or buckling.

## Serviceability limit states (SLS)

- Limit state of internal stress
  - The internal stresses developed in the materials of structural elements shall not exceed the specified magnitudes when subjected to combination of serviceability design actions.
  - The stresses are to be estimated using resistance models to represent the behaviour of structure, as stipulated in the Code.

# Limit States

## Serviceability limit states (SLS)

- Limit state of crack control
  - The cracking of reinforced, partially prestressed, and prestressed concrete structures under serviceability load combinations is kept within acceptable limits of crack widths in such a way as not to adversely affect the durability or impair the aesthetics.
  - Alternatively, the control of cracking is deemed to be satisfied by following restrictions on amount and spacing of reinforcement.

## Serviceability limit states (SLS)

- Limit state of deformation
  - The defonnation of the bridge or its elements when subjected to combination of design actions shall not adversely affect the proper functioning of its elements, appurtenances, and ridingquality.
  - Deformations during construction shall be controlled to achieve proper geometry of finished structure.

# Limit States

## Serviceability limit states (SLS)

- Limit state of vibration
  - For footbridges or component of bridges specifically designed to carry footway loading, the direct verification of vibration limits is required, for which specialist literature may be referred.
  - For special types of bridges and their components dynamic effects under action of wind are required to be calculated and verified to be within acceptable limits. Model tests are required under certain circumstances.
  - For other types of bridges, the limit state of vibration under serviceability load combinations is deemed to be satisfied by limiting deflection of elements.

# Limit States

## Serviceability limit states (SLS)

- Limit state of fatigue
  - The bridge or any of its components shall not lose its capacity to carry design loads by virtue of its materials reaching fatigue limits due to its loading history. For carrying out fatigue verification, specialist literature may be referred.



## Limit State of Collapse : Flexure

# Limit State of Collapse : Flexure

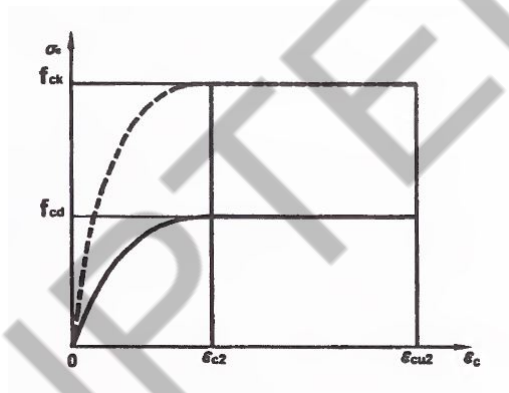
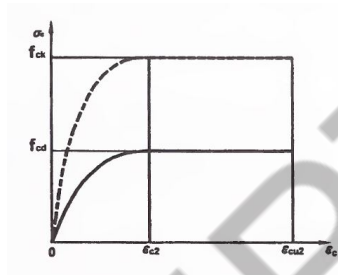


Figure 1: Parabolic-rectangular diagram for concrete in compression

# Limit State of Collapse : Flexure



$$f_{cd} = \frac{\alpha f_{ck}}{\gamma_m} \quad (1)$$

- where  $\alpha=0.67$
- $\gamma_m=1.5$  for basic and seismic combination
- $\gamma_m=1.2$  for accidental combination

Figure 2: Parabolic-rectangular diagram for concrete in compression

# Limit State of Collapse : Flexure

## Parabolic-rectangular stress block

- For design of section, the following relationship may be used

$$\sigma_c = f_{cd} \left[ 1 - \left( 1 - \frac{\epsilon_c}{\epsilon_{c2}} \right)^n \right] \quad \text{for } 0 \leq \epsilon_c \leq \epsilon_{c2} \quad (2)$$
$$\sigma_c = f_{cd} \quad \text{for } \epsilon_{c2} \leq \epsilon_c \leq \epsilon_{cu2}$$

- Upto concrete grade : M60
  - $n=2$
  - $\epsilon_{c2}=0.002$
  - $\epsilon_{cu2}=0.0035$

# Limit State of Collapse : Flexure

## **Simplified equivalent stress block**

- The parabolic rectangular stress-strain block is of general validity for all design situations.
- However, simplified equivalent stress blocks such as rectangle or bilinear may be used for design purposes where the net results are sufficiently accurate.

# Limit State of Collapse : Flexure

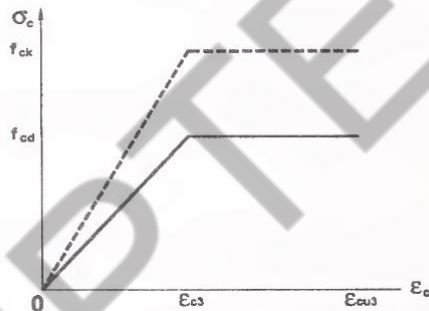


Figure 3: Bi-linear stress strain relation

# Limit State of Collapse : Flexure

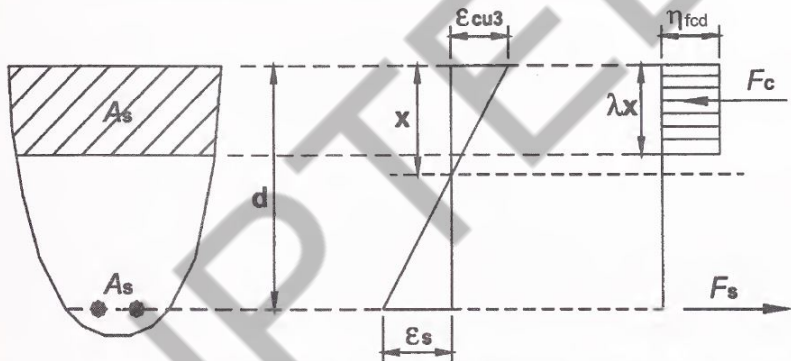


Figure 4: Rectangular stress distribution

# Limit State of Collapse : Flexure

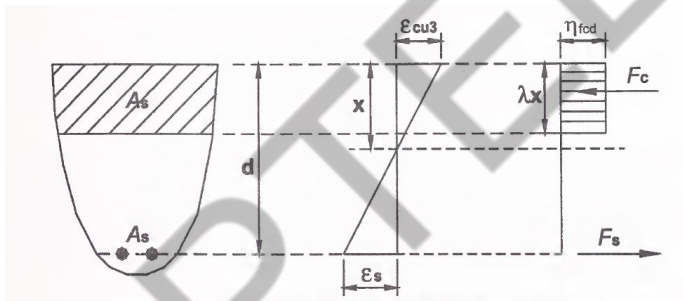


Figure 5: Rectangular stress distribution

- For  $f_{ck} \leq 60$  MPa :  $\lambda=0.8$ ,  $\eta=1$  and  $f_{cd} = \frac{4}{9}f_{ck}=0.445$



# Summary

# Summary

- Limit State Method presented in IRC 112:2011 is followed
- Basic design principles and different limit states are discussed
- Limit state of design for flexure is explained
- Also a comparison between working stress method and limit state method is also made

## References

# References

- IRC 21 : 2000** Standard specifications and code of practice for road bridges, Section III : Cement concrete (plain and reinforced) (Indian Roads Congress, New Delhi)
- IRC 112 : 2011** Code of practice for concrete road bridges (Indian Roads Congress, New Delhi)
- IS 456 : 2000** Indian Standard Plain and Reinforced Concrete (Bureau of Indian Standards, New Delhi)

Thank you